Mathematical Techniques to Improve the Utility of a Hazard Risk Matrix

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Abstract

This paper addresses how to increase the utility of a hazard risk matrix with techniques that leverage the mathematical relationship of severity, probability and risk on a well-defined matrix. It addresses the attributes of a well-defined matrix and how other quantifiable data can be integrated into a matrix to produce more meaningful measures and impacts of risk over the life-cycle of the system. Other techniques in this paper yield confidence that assessments of risk approximate reality by enabling comparison of risk assessments to actual accident data of a system and its legacy systems. The principles outlined in this paper should provide insights helpful for any practitioner applying a mishap risk assessment matrix to a specific system.

Introduction

While a risk assessment matrix is just one tool in the tool-box of a system safety practitioner it can be a useful one but only if the matrix is well-defined and the mathematical relationships associated with it are well understood. Numerous articles and papers have been written on risk assessment matrices including a few by this author (refs. 1-4).

Attributes of a Well-Designed Risk Assessment Matrix

An example of a well-defined risk assessment matrix is provided in Figure 1. This matrix was proposed in 2007 by the author for use as the DOD standard matrix for aviation and as one that could be adapted for any DOD system (ref.5). The attributes of a good risk assessment matrix were discussed in some detail in a panel discussion held at the 2004 International System Safety Conference and its results published in the Journal of System Safety in 2005 (ref.6). These attributes were included in ANSI/GEIA-STD-0010-2009 published in 2008 (ref.7).

The attributes of a well-defined matrix are numbered on Figure 1 as indicated in parentheses. (1) The first attribute of a well-defined matrix is that the severity scale should cover the full range of possible outcomes from potential accidents associated with the system. The example matrix could accommodate an extremely high value DOD system (up to $200 billion). (2) Next, probability, the likelihood of mishap occurrence, should be calibrated with reference to a standard or customer-defined exposure interval. Probability is, mathematically, a value between zero and one. Mishap frequency, the rate of mishap occurrence, is sometimes substituted for probability as a component of risk. (3) The severity and probability should be on equally-proportioned, logarithmic scales (1, 10, 100, 1000…) for reasons that are clear when considering attributes 5 and 8. (4) Next, the severity and probability (or frequency) axes should be oriented so that one axis increases upward and the other increases to the right in accordance with the Cartesian coordinate system making the orientation of the matrix consistent with standard engineering practice. (5) Risk levels should be assigned to cells consistent with contours of equal risk (iso-risk contours) which, because of attribute 3, are diagonals on the matrix. (6) Sufficient probability or frequency categories should be used so that the highest severity level hazards can be assessed at the lowest level of risk if probability can be reduced to an appropriately small number. (7) There should be a risk assessment code for hazards whose risk has been eliminated. (8) The matrix should be "tailorable" so that if the range of possible severity outcomes for a system is smaller than that provided by the matrix, severity and probability categories can be removed from the larger matrix. The severity and probability can then use the same designators, so, for example, a 5E, serious risk, would still be a 5E, serious risk, on a tailored, smaller matrix, or even a tailored, larger matrix. What enables the tailorability of this proposed matrix is that the numbering of the severity scale has been reversed from the MIL-STD-882D severity scale so that increasing severity numbers align with increasing severity. Otherwise, any increase or decrease in the range of severity requires renumbering the severity scale each time and the consistent designation of matrix cells is lost. The merits of well-defined matrix are illustrated by the tailoring of the matrix in Figure 1 to accommodate the entire planet Earth (Figure 2). A specific hazard, "Earth collision with an asteroid," is plotted and can be compared to other risks of interest to "spaceship" Earth.

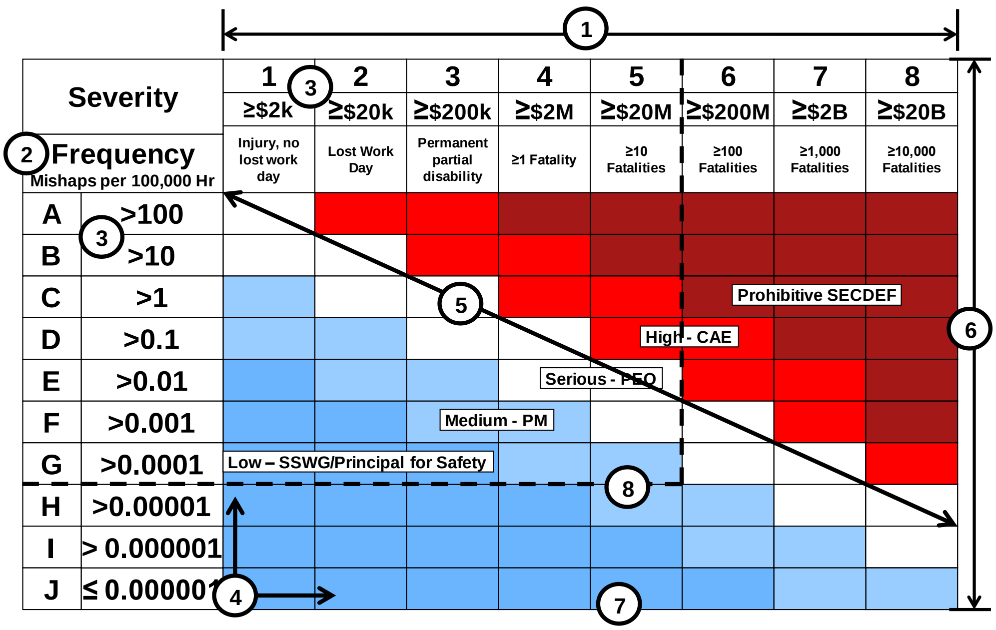

Figure 1 — A Well-Designed Risk Assessment Matrix

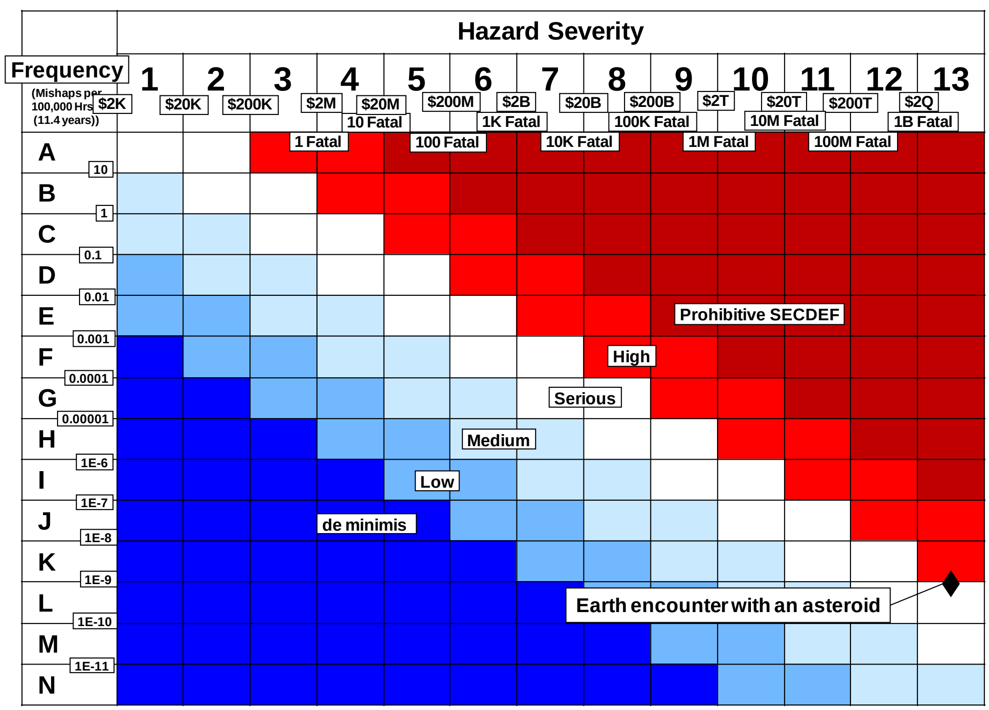

Figure 2 — The Mother of All Risk Assessment Matrices (Spaceship Earth)

<u>Understanding Probability</u>

Probability is defined by the American Heritage Dictionary of the English Language as “A number expressing the likelihood that a specific event will occur, expressed as the ratio of the number of actual occurrences to the number of possible occurrences.” A mathematical definition is as follows: Repeat a random experiment “n” number of times. If a specific outcome has occurred “f” times in these n trials, the number “f” is the frequency of the outcome. The ratio f/n is the relative frequency of the outcome. A relative frequency is usually very unstable for small values of “n,” but it tends to stabilize about some number “p” as “n” increases. The number “p” is the probability of the outcome defined by equation 1.

$$p = f / n \text{ (for very large values of n)} \quad (1)$$

In a simple example using a single die, what is the probability of rolling a 3? After one roll, a 5, f/n equals 0. The next roll, a 2, yields an f/n of 0. The third roll is a 3 and f/n now equals 1/3 or 0.333… The fourth roll, a "4," yields f/n equals 1/4 or 0.25. After 1000 rolls, we have 163 "3s" so f/n equals 163/1000 or 0.163. As the rolls approach infinity f/n equals 0.16666…This is the probability of rolling a "3" with one die. Table 1 shows a similar example but in this case the question is what is the probability that a military helicopter striking a wire resulting in a Class A mishap (death or loss of aircraft). Analysis has determined that the probability (actually frequency of occurrence) of this is 4.406E-06 occurrences per flight hour. Equipped with a grasp on good matrix attributes and probability, one can now apply this knowledge to expand the probability scale of the matrix.

Table 1 — Wire Strike Accidents

Accumulated Flight hours	Class A accidents	Accident Rate (per flight hour)
1	0	0
1,000	0	0
176,182	1	5.676E-06
274,539	2	7.285E-06
700,462	3	4.283E-06
10,000,000	46	4.600E-06
1,000,000,000	4407	4.407E-06
Approaches infinity		4.406E-06

Expanding the Matrix

Figure 3 is a version of the matrix used by the U.S. Army's Program Executive Officer for Aviation (PEO Aviation).

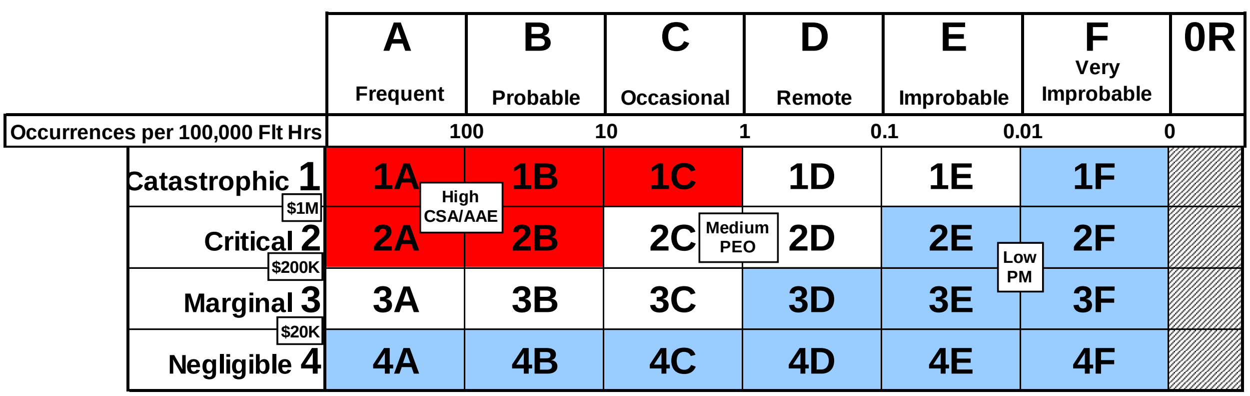

Figure 3 — PEO Aviation Matrix

It does not have all the attributes of a well-designed matrix but it does have attributes 1, 2, 3, 6 and 7. Column F, "Very Improbable" was added by the author to achieve attribute 6. For a specific aircraft acquisition program the fleet size will be 368 aircraft, each aircraft is projected to be flown 240 hours per year and each aircraft will be used for a total of 20 years. Using these numbers (eqns. 2-4) we can gain insight into this aircraft's exposure to risk.

$$\text{Aircraft Exposure} = 240 \text{ hours/year} \times 20 \text{ years} = 4{,}800 \text{ hours} \quad (2)$$

$$\text{Fleet Exposure} = 368 \text{ aircraft} \times 240 \text{ hours/year} \times 20\text{years} = 1{,}776{,}400 \text{ hours} \quad (3)$$

$$\text{Fleet Hours per Year} = 368 \text{ aircraft} \times 240 \text{ hours/year} = 88{,}320 \text{ hours/aircraft} \quad (4)$$

We can apply these values to expand the PEO Aviation matrix as shown in Figure 4. The rows "Flight Hours per Occurrence" and "Occurrences per Flight Hour are simply derived from "Occurrences per 100,000 Flt Hrs." The rows under "Fleet Life," "Occurrences per Year" are derived using equation 5 and "Years per Occurrence" is simply the inverse (1/X) of Occurrences per Year. The row Occurrence per Fleet Life is calculated using equation 6

$$88{,}320 \text{ flt hrs/year} \times 10 \text{ Occurrences } /100{,}000 \text{ flt hours} = 8.83 \text{ Occurrences/year} \quad (5)$$

$$1{,}776{,}400 \text{ hours/fleet life} \times 10 \text{ Occurrences}/100{,}000 \text{ flt hours} = 177 \text{ Occurrences/Fleet life} \quad (6)$$

The reason that both an item and its inverse are calculated is that one of them will yield a value greater than one. Values greater than one are easier for some people (the author included) to grasp than numbers expressed as decimals less than one. For example, 0.00883 occurrences per year means little but 113 years per occurrence gives a better sense of how very improbable the probability category of "F" is as applied to this specific fleet of aircraft.

The matrix in Figure 4 is actually a spreadsheet. Fleet size, Utilization, and Aircraft Life are input values along with Occurrences per 100,000 Flt Hrs. The spreadsheet calculates the remaining values. Once constructed, this expanded matrix allows anyone trying to comprehend probability to select the row with the most meaningful values. In addition, when communicating the "consequences of risk acceptance" in a risk acceptance document, this expanded matrix enables one to determine and provide to the risk acceptance authority that, for example, a 1D hazard will produce "on the order of 2 to 17 Class A accidents over the life cycle of this aircraft."

Assumptions

Fleet Size:	368 aircraft	Aircraft Exposure Hours:	4,800 hours
Utilization:	240 hours/yr	Fleet Exposure Hours:	1,766,400 hours
Aircraft Life:	20 years	Fleet Hours per Year:	88,320 hours

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable	0R
Occurrences per 100,000 Flt Hrs	100	10	1	0.1	0.01	0	
Flight Hours per Occurrence	1,000	10,000	100,000	1,000,000	10,000,000		
Occurrences per Flight Hour	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	0	
Fleet Life							
Occurrences per Year	**88.3**	**8.83**	0.883	0.0883	0.00883	0	
Years per Occurrence	0.0113	0.113	**1.13**	**11.3**	**113**		
Occurrence per Fleet Life	**1,766**	**177**	**17.7**	**1.77**	0.177	0	
Fleet Life per Occurrence	0.000566	0.00566	0.0566	0.566	**5.66**		
Catastrophic 1 ($1M)	1A	1B	1C	1D	1E	1F	
Critical 2 ($200K)	2A	2B	2C	2D	2E	2F	
Marginal 3 ($20K)	3A	3B	3C	3D	3E	3F	
Negligible 4	4A	4B	4C	4D	4E	4F	

High CSA/AAE

Medium PEO

Low PM

Figure 4 — Expanded PEO Aviation Matrix

Plotting Accidents on a Matrix

A fundamental principle of system safety is that safety risk over time results in accident loss. This means that an indicator of future risk is past accident loss rates.

Using accident data from the U.S. Army's Risk Management Information System (ref. 8), one can compute a "severity" (average Class A accident cost) from the cost and number of Class A accidents using equation 7.

$$\textbf{Severity} = \frac{\textbf{Total Cost of Class A mishaps}}{\textbf{Total Number of Class A mishaps}} = \frac{\textbf{\$361,671,038}}{\textbf{59}} = \textbf{\$6,130,018} \tag{7}$$

Next one can compute a frequency of occurrence (corresponding to probability) using the total number of Class A accidents and the total hours flown by the helicopter fleet using equation 8.

$$\textbf{Probability} = \frac{\textbf{Total Number of Class A mishaps}}{\textbf{Total Hours Flown}} = \frac{\textbf{59}}{\textbf{1,588,597}} = \textbf{3.714 mishaps / 100,000 Flt Hrs} \tag{8}$$

If we repeat this for Class B, C, and D accidents we get Table 2.

Table 2 — Aircraft Accident Data

Class	No	Total Cost	Cost/Mishap	Mishaps per 100,000 Flt Hrs
A	59	$361,671,038	$6,130,018	3.714
B	39	$18,854,121	$483,439	2.455
C	245	$17,114,206	$69,854	15.422
D	112	$970,148	$8,662	7.050

Then we can plot the last two columns of table 1 on logarithmic scales to get Figure 5.

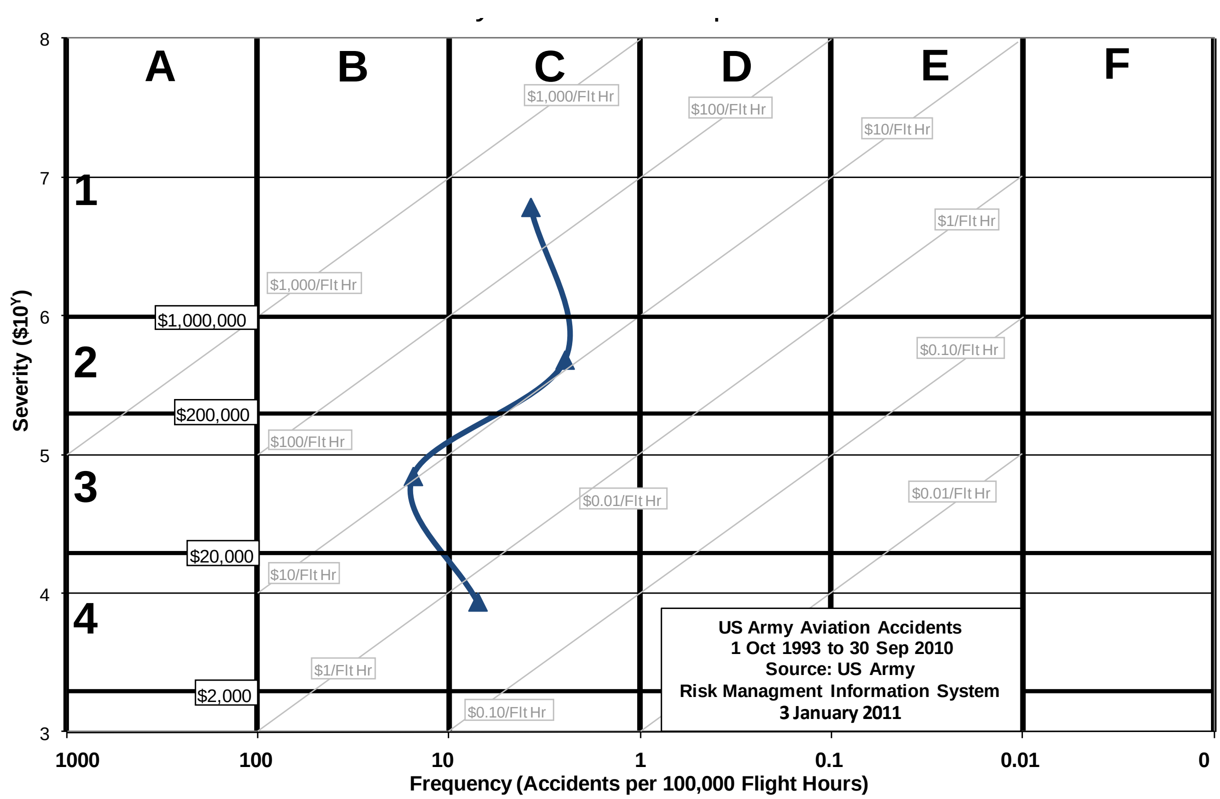

Figure 5 — Aircraft Accidents Plotted on Logarithmic Scales

Since accidents result from risk over time then Figure 5 represents graphically the total impact of the risk of all aircraft hazards to date. Figure 6 uses this same method to plot all hazards associated with actual Class A, B and C accidents for this aircraft type. Each point represents either a hazard or an accumulation of hazards that together sum to the Class A, B, C, and D points on the risk curve.

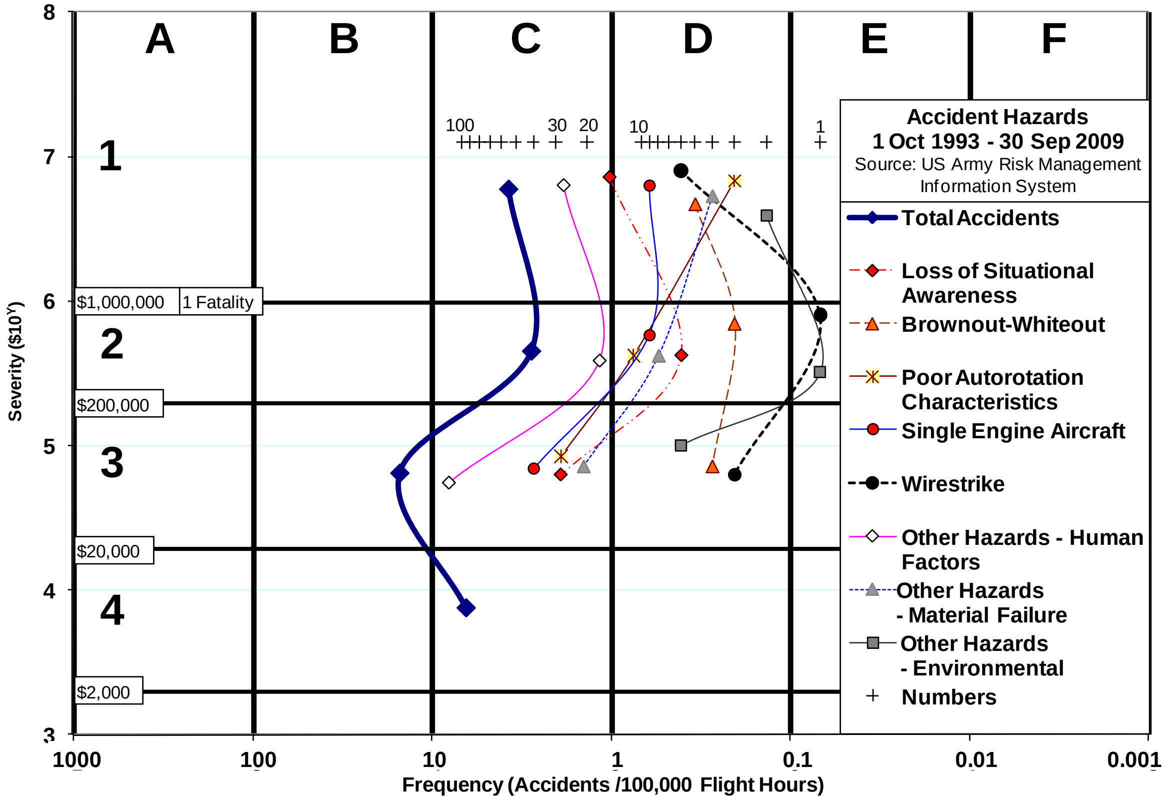

Figure 6 — Aircraft Accidents and Hazards Plotted on Logarithmic Scales

Relative Risk Values

Another mathematical technique that can be helpful is computing relative risk values. Table 3 shows a simplified matrix with the cell of the matrix with the lowest risk, 4F, assigned a risk value of 1. The author has named this single unit of risk a "Clemens" in honor of renowned system safety practitioner, Pat Clemens, author and co-author of numerous publications on system safety including many on the proper application of risk assessment matrices. The risk value of each cell of the matrix is 10 times the level of the cell below and to the right of it, as shown.

Table 3 — Relative Risk Values (Clemens)

		Probability					
		A	B	C	D	E	F
Severity	1	100,000,000	10,000,000	1,000,000	100,000	10,000	1,000
	2	10,000,000	1,000,000	100,000	10,000	1,000	100
	3	1,000,000	100,000	10,000	1,000	100	10
	4	100,000	10,000	1,000	100	10	1

Table 4 is the number of hazards for a specific Army helicopter program, identified here as Helicopter A. Each cell of the matrix contains the number of hazards for each risk assessment code (1D, 1E, 1F, etc.). Table 5 shows the product of the number of hazards times the relative risk value for the same cell. Figure 7 displays these values in a 3D column chart and yields a visual representation of the relative risk of hazards for each cell of the matrix. One can also compare the relative risk of similar systems. In this case Helicopter A is an aircraft being designed for a US ally similar to Helicopter B which is a legacy aircraft in the inventory for decades. Helicopter C is a new US aircraft that was designed to replace Helicopter B. Helicopter B is the aircraft whose accidents are plotted in Figure 6.

Table 4 — Helicopter A Hazard Distribution

		Probability					
		A	B	C	D	E	F
Severity	1				5	14	65
	2				4	6	2
	3			1	7	5	4
	4				2	1	

Table 5 — Helicopter A Relative Risk Distribution (Clemens)

		Probability					
		A	B	C	D	E	F
Severity	1				500,000	140,000	65,000
	2				40,000	6,000	200
	3			10,000	7,000	500	40
	4				200	10	

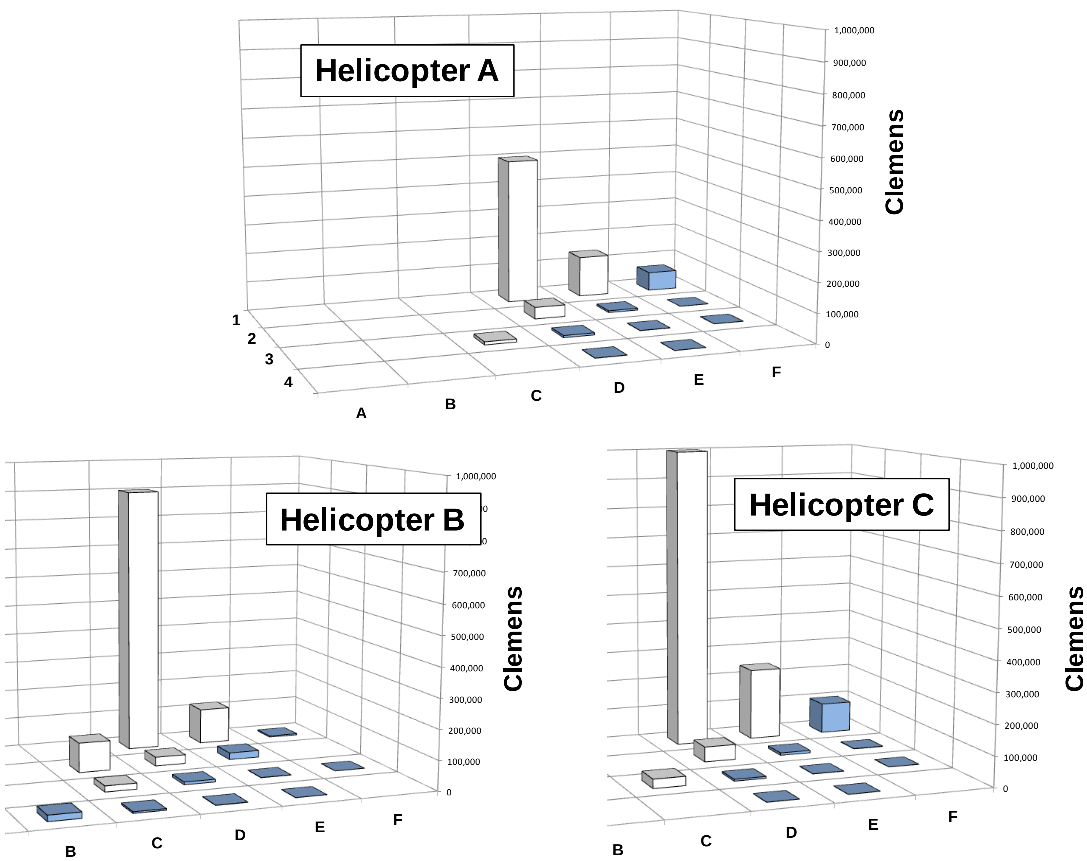

Figure 7 — Helicopter A, B, and C Relative Risk Distribution 3D Column Charts

The Helicopter A relative risk distribution in Table 5 can also be displayed in a pie chart to visualize the relative risk impact of the hazards in each cell (Figure 8). Here we see each of the five 1D hazards creates 13 percent of the risk for a total of 65 percent of the entire risk, while the 14 1E hazards create only 18 percent of the risk and the 65 1F hazards produce 8.5 percent of the risk.

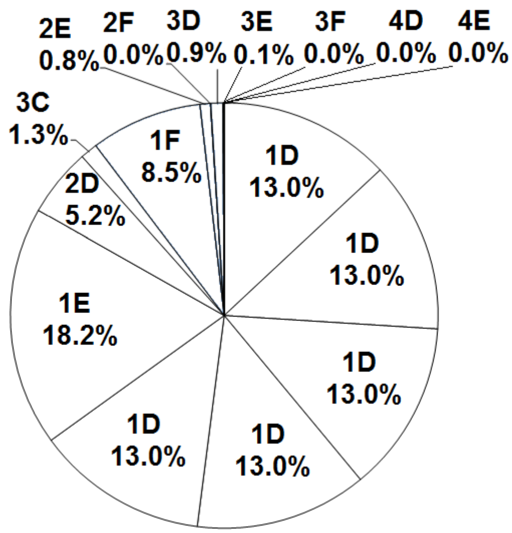

Figure 8 — Helicopter A Relative Risk Distribution Pie Chart

<u>Hazard Risk Profile</u>

The Helicopter A hazard distribution in Table 4 can also be used to produce a hazard risk profile similar in structure to the aircraft accidents plots in Figures 5 and 6. In Table 6 the values above each probability level is the value for the logarithmic center for each column. This number times the number of hazards produces a probability for that cell. They are then summed for each severity level (1, 2, 3, 4).

Table 6 — Helicopter A Hazard Risk Profile Computations

		Probability					
		3.16E-03	**3.16E-04**	**3.16E-05**	**3.16E-06**	**3.16E-07**	**3.16E-08**
		A	**B**	**C**	**D**	**E**	**F**
Severity	**1**	**Sum = 2.23E-05**			**5 x 3.16E-06 = 1.58E-05**	**14 x 3.16E-07 = 4.43E-06**	**65 x 3.16E-08 = 2.06E-06**
	2	**Sum = 1.46E-05**			**4 x 3.16E-06 = 1.26E-05**	**6 x 3.16E-07 = 1.90E-06**	**2 x 3.16E-08 = 6.32E-08**
	3	**Sum = 5.55E-05**		**1 x 3.16E-05 = 3.16E-05**	**7 x 3.16E-06 = 2.21E-05**	**6 x 3.16E-07 = 1.90E-06**	**2 x 3.16E-08 = 6.32E-08**
	4	**Sum = 6.64E-06**			**2 x 3.16E-06 = 6.32E-06**	**1 x 3.16E-07 = 3.16E-07**	

These values can be plotted on a matrix to produce the Helicopter A plot in Figure 9. This calculation can also be done for Helicopter B and C hazards and compared to the accident history plot for Helicopter B. Of note on this plot is how closely aligned the Helicopter C hazards are to the Helicopter B accident history. It only seriously diverges at Severity Level 4. This divergence may be due to the fact that hazards with severity above Severity Level 4 also produce accidents at that level that are not accounted for. It may also be due to the practitioner paying less attention to identifying hazards at the Severity 4 level because of focus on Severity 1, 2, and 3. Also note the Helicopter B's Hazard profile shows Severity level 4 hazards much greater than its own accident history. This is due to two hazards that had been assessed as 4B and five hazards assessed as 4C which should be reassessed since the accident history shows that they did not occur as often as predicted.

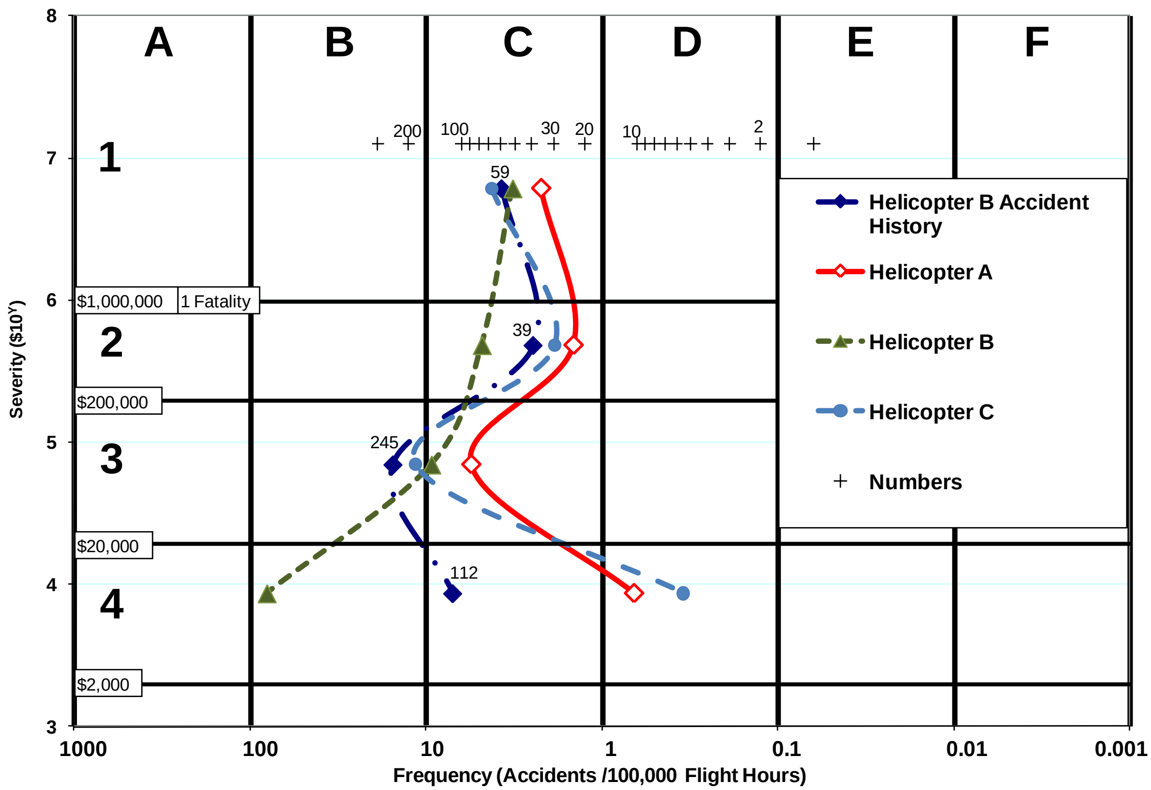

Figure 9 — Helicopter A Hazard Risk Profile

Conclusions

Bear in mind that while these techniques do not yield a high degree of precision, they are tools that facilitate getting hazards assessed to the appropriate cell of the matrix and can give confidence that the overall assessment of risk is somewhere close to reality. The risk assessment matrix is just one of many tools for managing system safety risk but these techniques for optimizing its utility can be very useful for those programs which have reasonably good accident data for analysis and have developed a risk assessment matrix that has most of the attributes of a well-designed matrix.

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Biography

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