		Severity			
		Catastrophic	Critical	Marginal	Negligible
Probability		1	2	3	4
Frequent	A				
Probable	B				
Occasional	C				
Remote	D				
Improbable	E				

Mathematical Techniques to Improve the Utility of a Hazard Risk Matrix

Don Swallom
U.S. Army Aviation and Missile Command
Redstone Arsenal, Alabama

Attributes of a well-designed risk assessment matrix

Severity scale covers full range of possible outcomes

Severity		1	2	3	4	5	6	7	8
		≥$2k	≥$20k	≥$200k	≥$2M	≥$20M	≥$200M	≥$2B	≥$20B
Frequency		Injury, no lost work day	Lost Work Day	Permanent partial disability	≥1 Fatality	≥10 Fatalities	≥100 Fatalities	≥1,000 Fatalities	≥10,000 Fatalities
A	>100								
B	>10								
C	>1								
D	>0.1								
E	>0.01								
F	>0.001								
G	>0.0001								
H	>0.00001								
I	> 0.000001								
J	≤ 0.000001								

Low – SSWG/Principal for Safety

Medium - PM

Serious - PEO

High - CAE

Prohibitive SECDEF

Proposed DOD Matrix

Attributes of a well-designed risk assessment matrix

Severity		1	2	3	4	5	6	7	8
		≥$2k	≥$20k	≥$200k	≥$2M	≥$20M	≥$200M	≥$2B	≥$20B
Frequency		Injury, no		Permanent					
A	>100								
B	>10								
C	>1								
D	>0.1								
E	>0.01								
F	>0.001								
G	>0.0001								
H	>0.00001								
I	> 0.000001								
J	≤ 0.000001								

Probability calibrated with reference to an exposure interval (accidents per 1,000 troops per year, accidents per 100,000 FH, accidents per 1,000,000 missile firings, etc.)

Prohibitive SECDEF

High - CAE

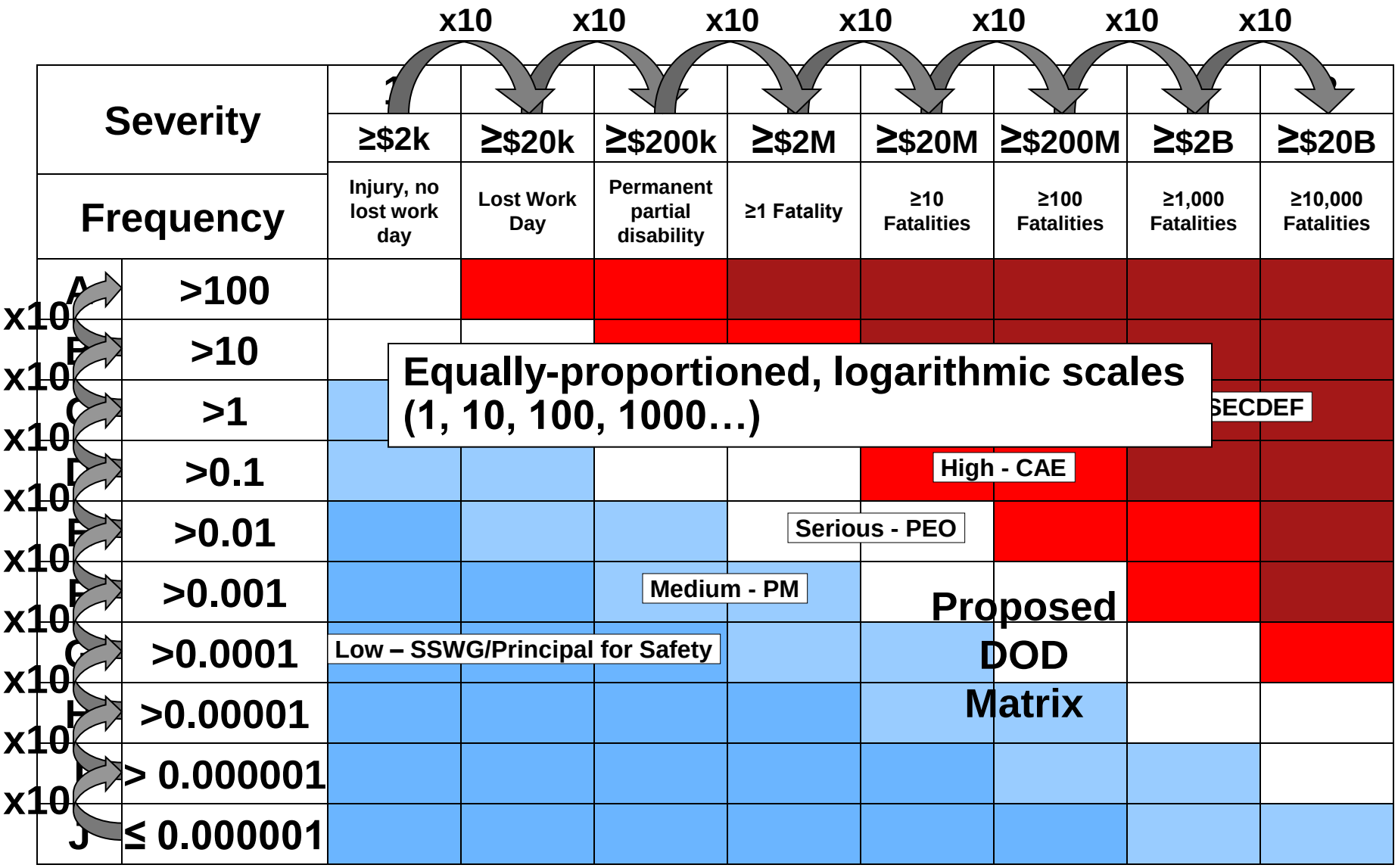
Serious - PEO

Medium - PM

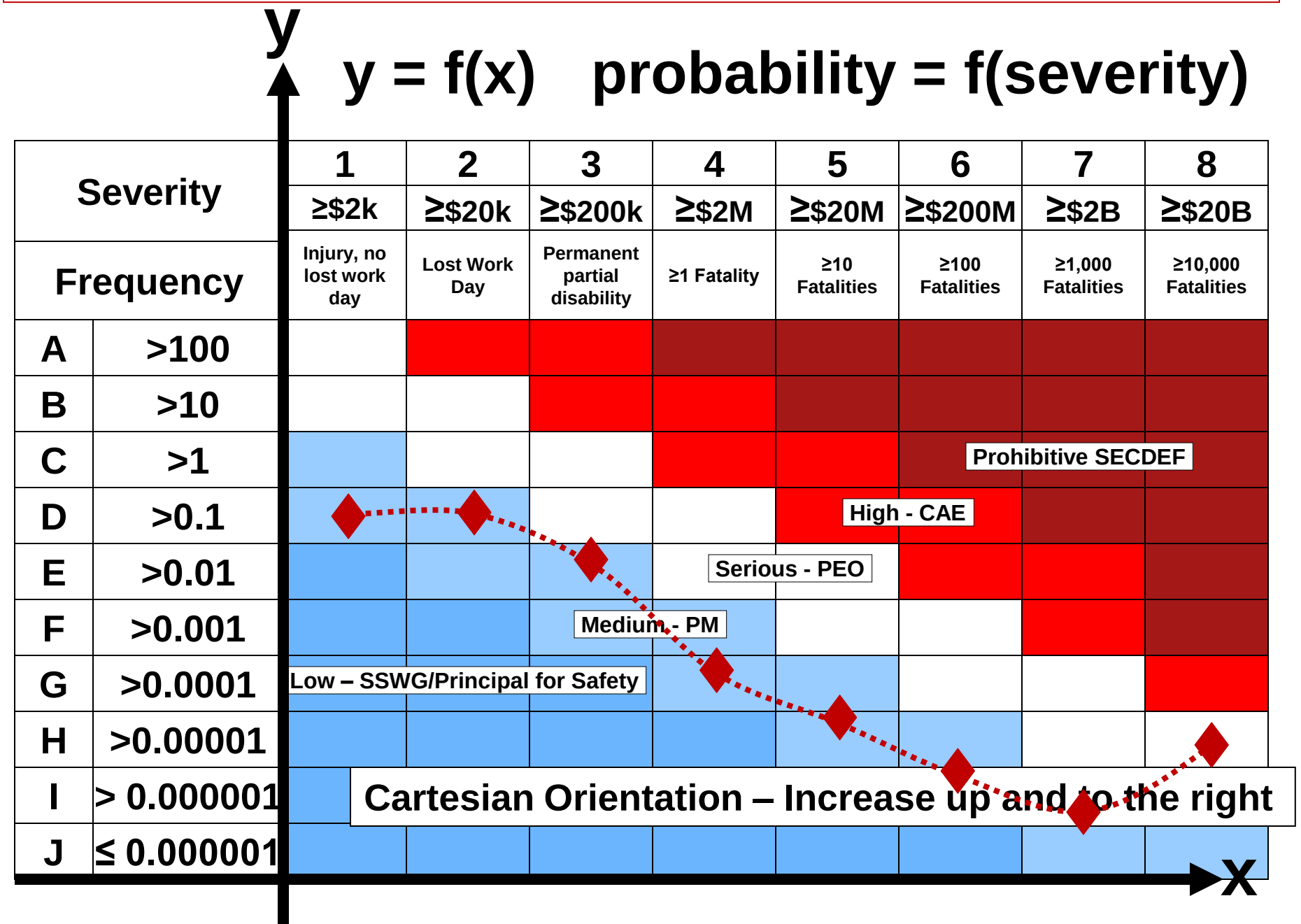
Low – SSWG/Principal for Safety

Proposed DOD Matrix

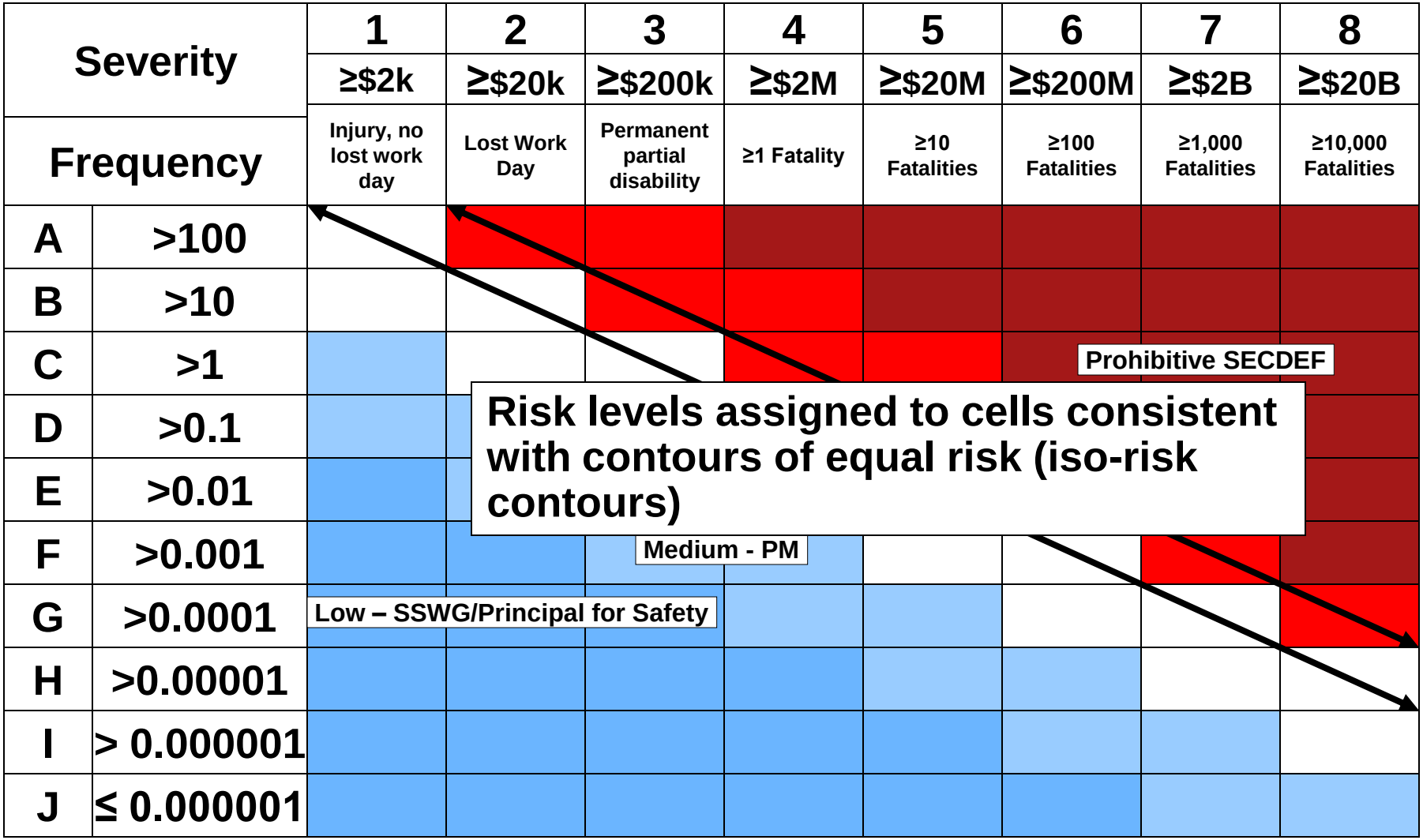
Attributes of a well-designed risk assessment matrix



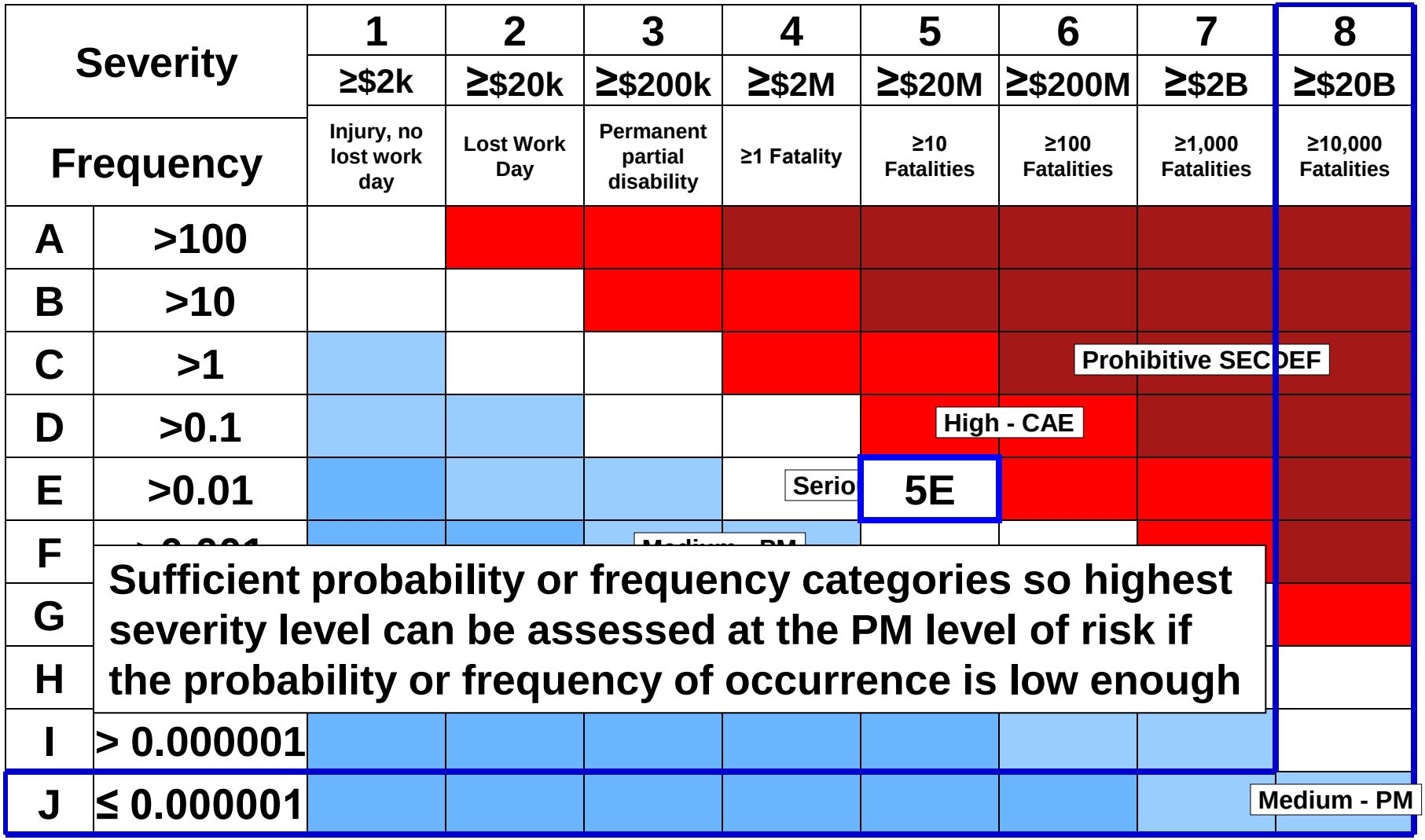
Attributes of a well-designed risk assessment matrix



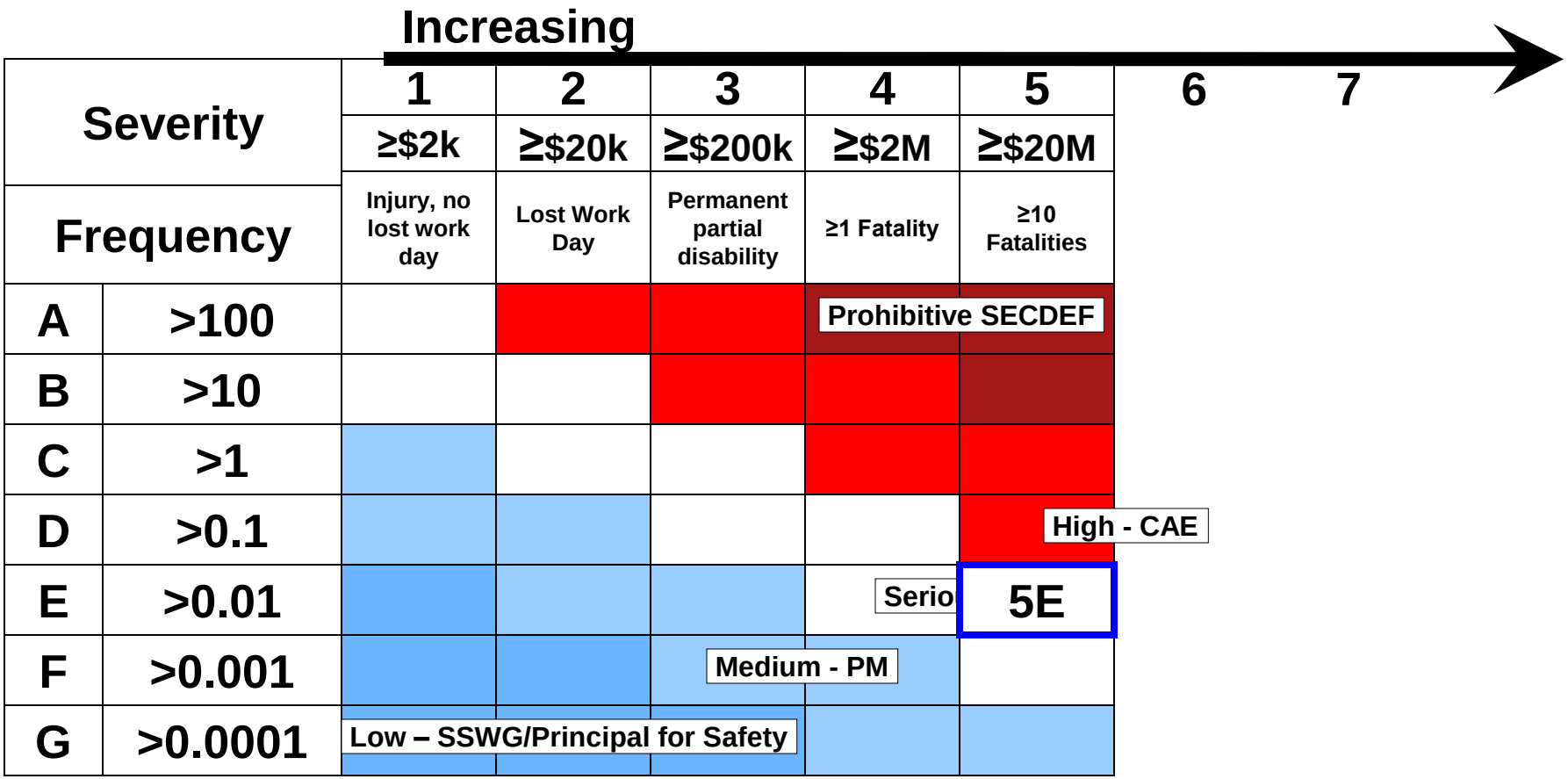
Attributes of a well-designed risk assessment matrix



Attributes of a well-designed risk assessment matrix

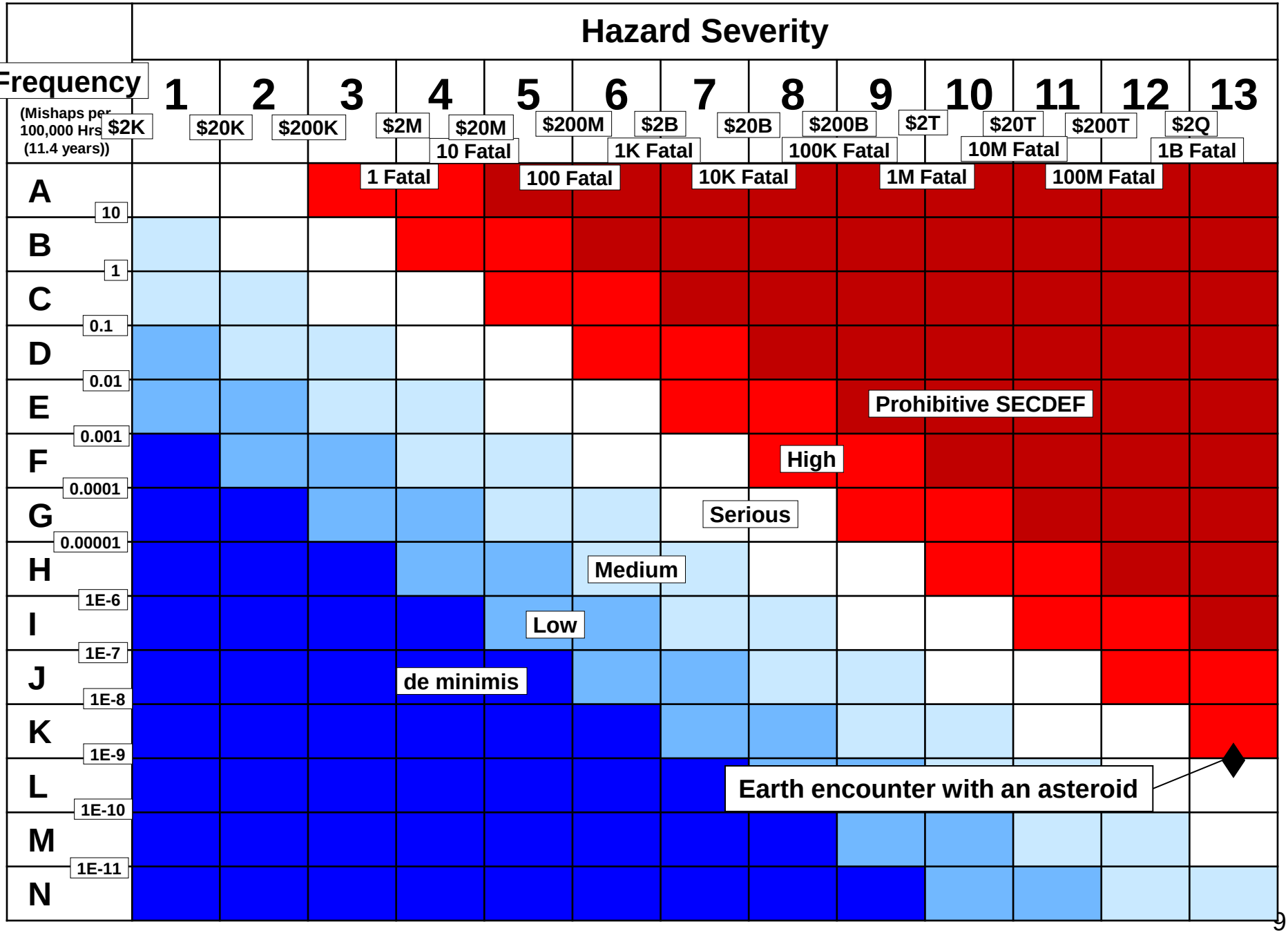


Attributes of a well-designed risk assessment matrix



Easily tailored with reporting of risk consistent with other systems within the family of systems.

Mother of All Risk Assessment Matrices (Spaceship Earth)



U.S. Army System Safety Risk Matrix (MIL-STD-882D)

RISK Acceptance Levels per DODI 5000.02, 8 Dec 08 Risk Assessment Levels & Definitions per Tables A-1 thru A-IV of MIL-STD 882D, 10 Feb 00			HAZARD SEVERITY			
			Catastrophic	Critical	Marginal	Negligible
Specific Individual Item	Fleet or Inventory		Could result in death, permanent total disability, loss exceeding $1M, or irreversible severe environmental damage that violates law or regulation.	Could result in permanent partial disability, injuries or occupational illness that may result in hospitalization of at least three personnel, loss exceeding $200K but less than $1M, or reversible environmental damage causing a violation of law or regulation.	Could result in injury or occupational illness resulting in one or more lost work days(s), loss exceeding $10K but less than $200K, or mitigatible environmental damage without violation of law or regulation where restoration activities can be accomplished.	Could result in injury or illness not resulting in a lost work day, loss exceeding $2K but less than $10K, or minimal environmental damage not violating law
			1	2	3	4
Likely to occur often in the life of an item, with a probability of occurrence greater than 10^{-1} in that life.	Continuously experienced	A	1A HIGH AAE	2A HIGH AAE	3A SERIOUS PEO	4A MEDIUM PM
Will occur several times in the life of an item, with a probability of occurrence less than 10^{-1} but greater than 10^{-2} in that life	Will occur frequently	B	1B HIGH AAE	2B HIGH AAE	3B SERIOUS PEO	4B MEDIUM PM
Likely to occur some time in the life of an item, with a probability of occurrence less than 10^{-2} but greater than 10^{-3} in that life	Will occur several times	C	1C HIGH AAE	2C SERIOUS PEO	3C MEDIUM PM	4C LOW PM
Unlikely but possible to occur in the life of an item, with a probability of occurrence less than 10^{-3} but greater than 10^{-6} in that life	Unlikely, but can reasonably be expected to occur	D	1D SERIOUS PEO	2D MEDIUM PM	3D MEDIUM PM	4D LOW PM
So unlikely, it can be assumed occurrence may not be experienced, with a probability of occurrence less than 10^{-6} in that life	Unlikely to occur, but possible	E	1E MEDIUM PM	2E MEDIUM PM	3E MEDIUM PM	4E LOW PM

U.S. Army System Safety Risk Matrix (MIL-STD-882D)

RISK Acceptance Levels per DODI 5000.02, 8 Dec 08
Risk Assessment Levels & Definitions per Tables A-1 thru A-IV of MIL-STD 882D, 10 Feb 00

Specific Individual Item	Fleet or Inventory		HAZARD SEVERITY: Catastrophic	Critical	Marginal	Negligible
			Could result in death, permanent total disability, loss exceeding $1M, or irreversible severe environmental damage that violates law or regulation.	Could result in permanent partial disability, injuries or occupational illness that may result in hospitalization of at least three personnel, loss exceeding $200K but less than $1M, or reversible environmental damage causing a violation of law or regulation.	Could result in injury or occupational illness resulting in one or more lost work days(s), loss exceeding $10K but less than $200K, or mitigated environmental damage without violation of law or regulation where restoration activities can be accomplished.	Could result in injury or illness not resulting in a lost work day, loss exceeding $2K but less than $10K, or minimal environmental damage without violation of law or regulation.
			1	2	3	4
Likely to occur often in the life of an item, with a probability of occurrence greater than 10^{-1} in that life	Continuously experienced	A	1A HIGH AAE	2A HIGH AAE	3A SERIOUS PEO	4A MEDIUM PM
Will occur frequently in the life of an item, with a probability of occurrence less than 10^{-1} but greater than 10^{-2} in that life	Will occur frequently	B	1B HIGH AAE	2B HIGH AAE	3B SERIOUS PEO	4B MEDIUM PM
Likely to occur some time in the life of an item, with a probability of occurrence less than 10^{-2} but greater than 10^{-3} in that life	Will occur several times	C	1C HIGH AAE	2C SERIOUS PEO	3C MEDIUM PM	4C LOW PM
Unlikely, but can reasonably be expected to occur, but probability of occurrence less than 10^{-3} but greater than 10^{-6} in that life	Unlikely, but can reasonably be expected to occur	D	1D SERIOUS PEO	2D MEDIUM PM	3D MEDIUM PM	4D LOW PM
So unlikely, it can be assumed occurrence may not be experienced, with a probability of occurrence less than 10^{-6} in that life	Unlikely to occur, but possible	E	1E MEDIUM PM	2E MEDIUM PM	3E MEDIUM PM	4E LOW PM

x10 10^{-1} x10 10^{-2} x1,000 10^{-3} 10^{-6}

x5 $1M x20 $200K x5 $10K $2K

U.S. Army System Safety Risk Matrix (MIL-STD-882E)

RISK Acceptance Levels per DODI 5000.02, 8 Dec 08 Risk Assessment Levels & Definitions per Tables A-1 thru A-IV of MIL-STD 882D, 10 Feb 00			HAZARD SEVERITY			
			Catastrophic	Critical	Marginal	Negligible
Specific Individual Item	Fleet or Inventory		Could result in one or more of the following: death, permanent total disability, irreversible significant environmental, impact , or monetary loss exceeding $10M.	Could result in one or more of the following: permanent partial disability, injuries or occupational illness that may result in hospitalization of at least t… perso… rev…sible s…nificant …vironmental …, or mo… …xceedi… but less than $10M.	Could result in one or more of the following: injury or occupational illness resulting in 10 or more lost work days, reversible mode… envir… impa… …ary lo… exceeding …00K but less tha… …1M.	Could result in one or more of the following: injury or illness resulting in less than 10 lost work days, minimal environmental impact or monetary loss … $100…
			1	2	3	4
Likely to occur often in the life of an item.	Continuously experienced.	A	1A HIGH AAE	2A HIGH AAE	3A HIGH AAE	4A SERIOUS PEO
Will occur sev… item.	…occur frequently	B	1B HIGH AAE	2B HIGH AAE	3B SERIOUS PEO	4B MEDIUM PM
Likely to occur some time in the life of an item.	Will occur several times	C	1C HIGH AAE	2C SERIOUS PEO	3C MEDIUM PM	4C MEDIUM PM
Unlikely but possible to occur in the life of an item.	Unlikely, but can reasonably be expected to occur	D	1D SERIOUS PEO	2D MEDIUM PM	3D MEDIUM PM	4D LOW PM
So unlikely, it can be assumed occurrence may not be experienced in the life of an item.	Unlikely to occur, but possible	E	1E MEDIUM PM	2E MEDIUM PM	3E LOW PM	4E LOW PM

x10 x10 x10

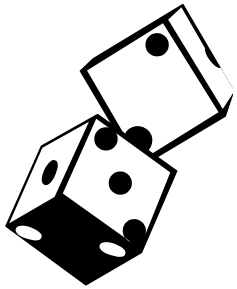
$10M $1M $100K $10K

No Numbers

U.S. Army System Safety Risk Matrix (MIL-STD-882D)

RISK Acceptance Levels per DODI 5000.02, 8 Dec 08 Risk Assessment Levels & Definitions per Tables A-1 thru A-IV of MIL-STD 882D, 10 Feb 00			HAZARD SEVERITY			
			Catastrophic	Critical	Marginal	Negligible
Specific Individual Item	Fleet or Inventory		Could result in one or more of the following: death, permanent total disability, irreversible significant environmental, impact , or monetary loss exceeding $10M.	Could result in one or more of the following: permanent partial disability, injuries or occupational illness that may result in hospitalization of at least three personnel, reversible significant environmental impact, or monetary loss exceeding $1M but less than $10M.	Could result in one or more of the following: injury or occupational illness resulting in 10 or more lost work days, reversible moderate environmental impact, monetary loss exceeding $100K but less than $1M.	Could result in one or more of the following: injury or illness resulting in less than 10 lost work days, minimal environmental impact, or monetary loss less than $100K.
			1	2	3	4
Likely to occur often in the life of an item.	Continuously experienced	A	1A HIGH AAE	2A HIGH AAE	3A SE[illegible]S P[illegible]	[illegible]M PM
Will occur several times in the life of an item.	Will occur frequently	B	1B HIGH AAE	2B HIGH AAE	3B SERIOUS PEO	4B MEDIUM PM
Likely to occur some time in the life of an item.	Will occur several times	C	1C HIGH AAE	2C SERIOUS PEO	3C MEDIUM PM	4C [illegible] PM
Unlikely but possible to occur in the life of an item.	Unlikely, but can reasonably be expected to occur	D	1D SERIOUS PEO	2D MEDIUM PM	3D MEDIUM PM	4D LOW PM
So unlikely, it can be assumed occurrence may not be experienced in the life of an item.	Unlikely to occur, but possible	E	1E MEDIUM PM	2E MEDIUM PM	3E [illegible]M PM	4E LOW PM

Understanding Probability

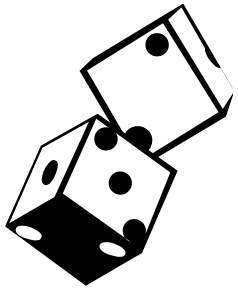


Probability:

“A number expressing the likelihood that a specific event will occur, expressed as the ratio of the number of actual occurrences to the number of possible occurrences.”

- The American Heritage® Dictionary of the English Language, Fourth Edition

Understanding Probability



Math Definition:

- **Repeat a random experiment “n” number of times.**
- **If a specific outcome has occurred “f” times in these n trials, the number “f” is the frequency of the outcome.**
- **The ratio f/n is the relative frequency of the outcome.**
- **A relative frequency is usually very unstable for small values of “n,” but it tends to stabilize about some number “p” as “n” increases.**
- **The number “p” is the probability of the outcome.**

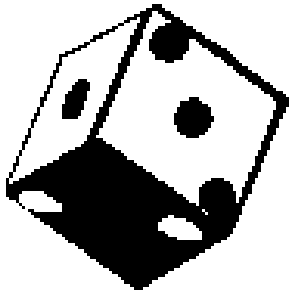
$$p = f / n$$

for very large values of n

Understanding Probability

Simple example:

Probability of rolling a “3” with one die.



Roll #1 - “5”, f/n = 0/1 = 0

Roll #2 - “2”, f/n = 0/2 = 0

Roll #3 - “3”, f/n = 1/3 = .333…

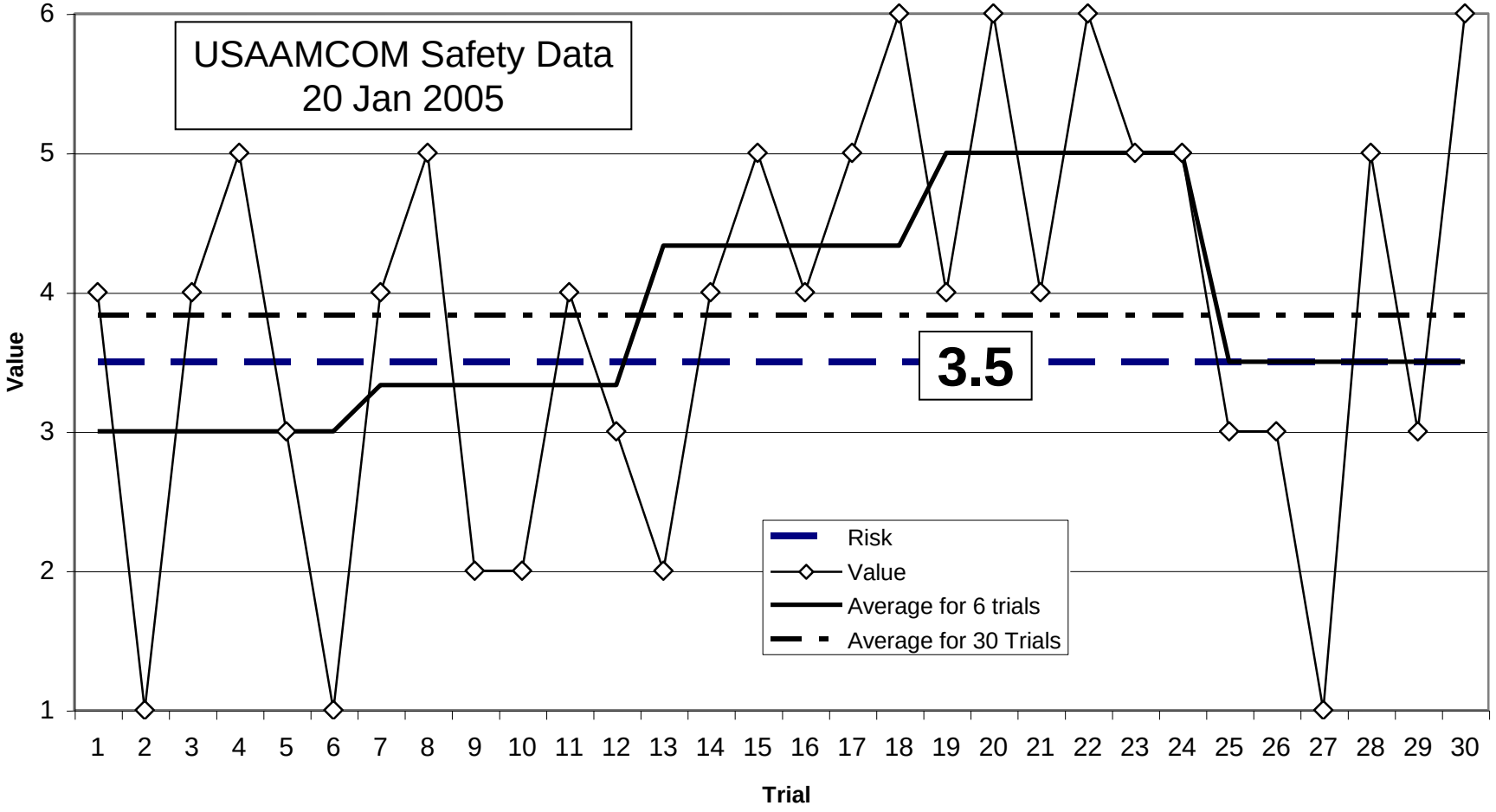
Roll #4 - “4”, f/n = 1/4 = .25

Roll #1,000: 163 “3”s, f/n = 163/1000 = .163

Rolls approach infinity f/n = .166666….

Rolling Dice

Roll a single die 30 times. The expected value of each roll is 3.5. What you actually get is somewhat different.



Understanding Probability

Hazard: Helicopter strikes wire; results in Class A mishap

Probability: 4.406E-06 occurrences per flight hour

1 Flight Hr, no mishap, rate = 0

1,000 Flight Hrs, no mishap, rate = 0

176,182 Flight Hrs, 1 mishap, rate = 5.676E-06 /flt hr

274,539 Flight Hrs, 2 mishaps, rate = 7.285E-06 /flt hr

700,462 Flt Hrs, 3 mishaps, rate = 4.283E-06 /flt hr

10,000,000 Flt Hrs, 46 mishaps, rate = 4.600E-06 /flt hr

1,000,000,000 Hrs, 4407 mishaps, rate = 4.407E-06 /flt hr

Flight hours approach infinity, rate = 4.406E-06 /flt hr

NAVAIR System Safety Risk Matrix

HAZARD CATEGORIZATION		SEVERITY			
		CATASTROPHIC (1)	CRITICAL (2)	MARGINAL (3)	NEGLIGIBLE (4)
FREQUENCY	FREQUENT (A) = or > 100/100K flt hrs	1	3	7	13
	PROBABLE (B) 10-99/100K flt hrs	2	5	9	16
	OCCASIONAL (C) 1.0-9.9/100K flt hrs	4	6	11	18
	REMOTE (D) 0.1-0.99/100K flt hrs	8	10	14	19
	IMPROBABLE (E) = or < 0.1/100K flt hrs	12	15	17	20

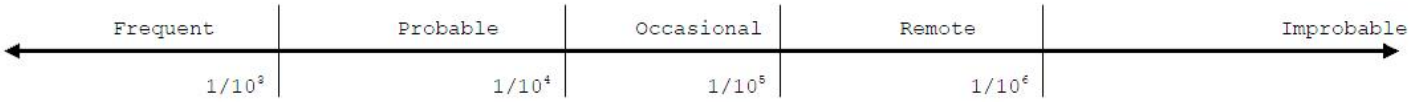
UNACCEPTABLE	ASN/CNO / CMC/CFFC* Acceptance 1-5 HIGH SAFETY RISK	ACCEPTABLE WITH REVIEW	PM/TYCOM N43/WING CDR* Acceptance 11-17 MEDIUM SAFETY RISK
UNDESIRABLE	PEO/CFFC N43/TYCOM* Acceptance 6-10 SERIOUS SAFETY RISK	ACCEPTABLE WITHOUT REVIEW	PM/RMB Acceptance 18-20 LOW SAFETY RISK

* Fleet Acceptance for aircraft that have achieved IOC

Severity is the worst credible consequence of a hazard in terms of degree of injury, property damage or effect on mission defined below:

Catastrophic - Class A (damage > $1M / fatality / permanent total disability)
Critical - Class B ($200K < damage < $1M / permanent partial disability / hospitalization of 3 or more personnel)
Marginal - Class C ($20K < damage < $200K / injury results in 1 or more lost workdays)
Negligible - All other injury/damage less than Class C

Probability of occurrence for discrete events may replace **Frequency** based upon the chart below:



NAVAIR System Safety Risk Matrix

FREQUENT (A) = or > 100/100K flt hrs	**100**
PROBABLE (B) 10-99/100K flt hrs	**10**
OCCASIONAL (C) 1.0-9.9/100K flt hrs	**1**
REMOTE (D) 0.1-0.99/100K flt hrs	**0.1**
IMPROBABLE (E) = or < 0.1/100K flt hrs	**0.01**

	SEVERITY		
	CRITICAL (2)	MARGINAL (3)	NEGLIGIBLE (4)
	3	7	13
	5	9	16
	6	11	18
	10	14	19
	15	17	20

ACCEPTABLE WITH REVIEW	PM/TYCOM N43/WING CDR* Acceptance 11-17 MEDIUM SAFETY RISK
ACCEPTABLE WITHOUT REVIEW	PM/RMB Acceptance 18-20 LOW SAFETY RISK

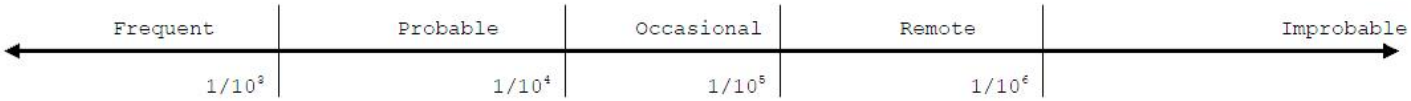
SK
ance for aircraft that have achieved IOC
ms of degree of injury, property damage or effect on mission defined below:

/ fatality / permanent total

1M / permanent partial disability / hospitalization of 3 or more personnel)
00K / injury results in 1 or more

ess than

lace **Frequency** based upon the chart below:



US Army PEO Aviation Matrix

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable	0R
Occurrences per 100,000 Flt Hrs	100	10	1	0.1	0.01	0	
Catastrophic 1 $1M	1A	1B	1C	1D	1E	1F	
Critical 2 $200K	2A	2B	2C	2D	2E	2F	
Marginal 3 $20K	3A	3B	3C	3D	3E	3F	
Negligible 4	4A	4B	4C	4D	4E	4F	

High CSA/AAE

Medium PEO

Low PM

Applying Probability Classifications to a military helicopter

Fleet Size = 368 aircraft

Utilization = 240 hours/year

Life = 20 years/aircraft

Aircraft Life = 240 x 20
= 4,800 hours

Fleet Exposure Hours = 368 x 240 x 20
= 1,776,400 hours

Fleet Hours per Year = 368 x 240
= 88,320 hours

US Army PEO Aviation Expanded Matrix

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable
Occurrences per 100,000 Flt Hrs:	**100**	**10**	**1**	**0.1**	**0.01**	**0**
Flight Hours per Occurrence:	**1,000**	**10,000**	**100,000**	**1,000,000**	**10,000,000**	
Occurrences per Flight Hour:	**10^{-3}**	**10^{-4}**	**10^{-5}**	**10^{-6}**	**10^{-7}**	**0**

88,320 flt hrs /year x 10 Occ/100,000 flt hours = 8.83 Occ/year

Occurrences per Year:	**88.3**	**8.83**	0.883	0.0883	0.00883	**0**
Years per Occurrence:	0.0113	0.113	**1.13**	**11.3**	**113**	

1,776,400 flt hrs / fleet life x 10 Occ/100,000 flt hours = 177 Occ/life

Occurrences per Fleet Life:	**1,766**	**177**	**17.7**	**1.77**	0.177	**0**
Fleet Life per Occurrence:	0.000566	0.00566	0.0566	0.566	**5.66**	

US Army PEO Aviation Expanded Matrix

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable
Occurrences per 100,000 Flt Hrs:	100	10	1	0.1	0.01	0
Flight Hours per Occurrence:	1,000	10,000	100,000	1,000,000	10,000,000	
Occurrences per Flight Hour:	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	0
Occurrences per Year:	**88.3**	**8.83**	0.883	0.0883	0.00883	**0**
Years per Occurrence:	0.0113	0.113	**1.13**	**11.3**	**113**	
Occurrences per Fleet Life:	**1,766**	**177**	**17.7**	**1.77**	0.177	**0**
Fleet Life per Occurrence:	0.000566	0.00566	0.0566	0.566	**5.66**	

Numbers greater then 1 are easier to understand and apply

US Army PEO Aviation Expanded Matrix

Input
Calculated

Assumptions

Fleet Size:	368	aircraft	
Utilization:	240	hours/yr	
Aircraft Life:	20	years	
Aircraft Exposure Hours:	4,800	hours	
Fleet Exposure Hours:	1,766,400	hours	
Fleet Hours per Year:	88,320	hours	

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable	0R
Occurrences per 100,000 Flt Hrs	100	10	1	0.1	0.01		0
Flight Hours per Occurrence	1,000	10,000	100,000	1,000,000	10,000,000		
Occurrences per Flight Hour	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}		0
Fleet Life							
Occurrences per Year	**88.3**	**8.83**	0.883	0.0883	0.00883		0
Years per Occurrence	0.0113	0.113	**1.13**	**11.3**	**113**		
Occurrence per Fleet Life	**1,766**	**177**	**17.7**	**1.77**	0.177		0
Fleet Life per Occurrence	0.000566	0.00566	0.0566	0.566	**5.66**		

Catastrophic 1 $1M	1A	1B	1C	1D	1E	1F	
Critical 2 $200K	2A	2B	2C	2D	2E	2F	
Marginal 3 $20K	3A	3B	3C	3D	3E	3F	
Negligible 4	4A	4B	4C	4D	4E	4F	

High CSA/AAE

Medium PEO

Low PM

US Army PEO Aviation Expanded Matrix

Assumptions

Fleet Size:	368 aircraft	Aircraft Exposure Hours:	4,800 hours
Utilization:	240 hours/yr	Fleet Exposure Hours:	1,766,400 hours
Aircraft Life:	20 years	Fleet Hours per Year:	88,320 hours

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable	0R
Occurrences per 100,000 Flt Hrs	100	10	1	0.1	0.01	0	
Flight Hours per Occurrence	1,000	10,000	100,000	1,000,000	10,000,000		
Occurrences per Flight Hour	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	0	
Fleet Life							
Occurrences per Year	**88.3**	**8.83**	0.883	0.0883	0.00883	**0**	
Years per Occurrence	0.0113	0.113	**1.13**	**11.3**	**113**		
Occurrence per Fleet Life	**1,766**	**177**	**17.7**	**1.77**	0.177	**0**	
Fleet Life per Occurrence	0.000566	0.00566	0.0566	0.566	**5.66**		

Catastrophic 1 $1M	1A	1B	1C	1D	1E	1F	
Critical 2 $200K	2A	2B	2C	2D	2E	2F	
Marginal 3 $20K	3A	3B	3C	3D	3E	3F	
Negligible 4	4A	4B	4C	4D	4E	4F	

High CSA/AAE

Medium PEO

Low PM

Consequences of Risk Acceptance

Assumptions

Fleet Size:	368 aircraft	Aircraft Exposure Hours:	4,800 hours
Utilization:	240 hours/yr	Fleet Exposure Hours:	1,766,400 hours
Aircraft Life:	20 years	Fleet Hours per Year:	88,320 hours

Consequences of Risk Acceptance:
On the order of 2 to 17 Class A accidents due to this hazard over the life cycle of the aircraft.

	A	B	C	D	E	F	0R
Occurrences per Flight Hour							
Fleet Life							
Occurrences per Year	88.3	8.83	0.883	0.0883	0.00883		0
Years per Occurrence	0.0113	0.113	1.13	11.3	113		
Occurrence per Fleet Life	1,766	177	17.7	17 - 2	1.77	0.177	0
Fleet Life per Occurrence	0.000566	0.00566	0.0566	0.566	5.66		

Catastrophic 1 $1M	1A	1B	1C	1D	1E	1F	
Critical 2 $200K	2A	2B	2C	2D	2E	2F	
Marginal 3 $20K	3A	3B	3C	3D	3E	3F	
Negligible 4	4A	4B	4C	4D	4E	4F	

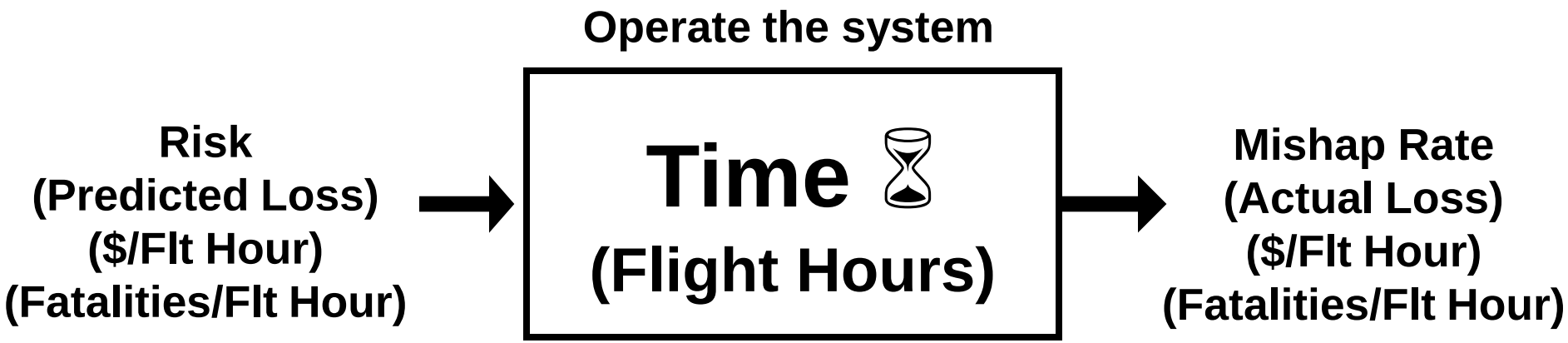
High CSA/AAE

Medium PEO

Low PM

Mishap Risk & Mishap Loss

Mishap Risk over Time results in Mishap Loss



Mishap History

Based on this relationship between mishap risk and mishap loss, we can plot mishap histories on a risk matrix as follows:

$$\text{Severity} = \frac{\text{Total Cost from Class A mishaps}}{\text{Total Number of Class A mishaps}}$$

$$= \frac{\$361,671,038}{59} = \$\ 6,130,018$$

$$\text{Probability} = \frac{\text{Total Number of Class A mishaps}}{\text{Total Hours Flown}}$$

$$= \frac{59}{1,588,597} = 3.714 \text{ mishaps / 100,000 Flt Hrs}$$

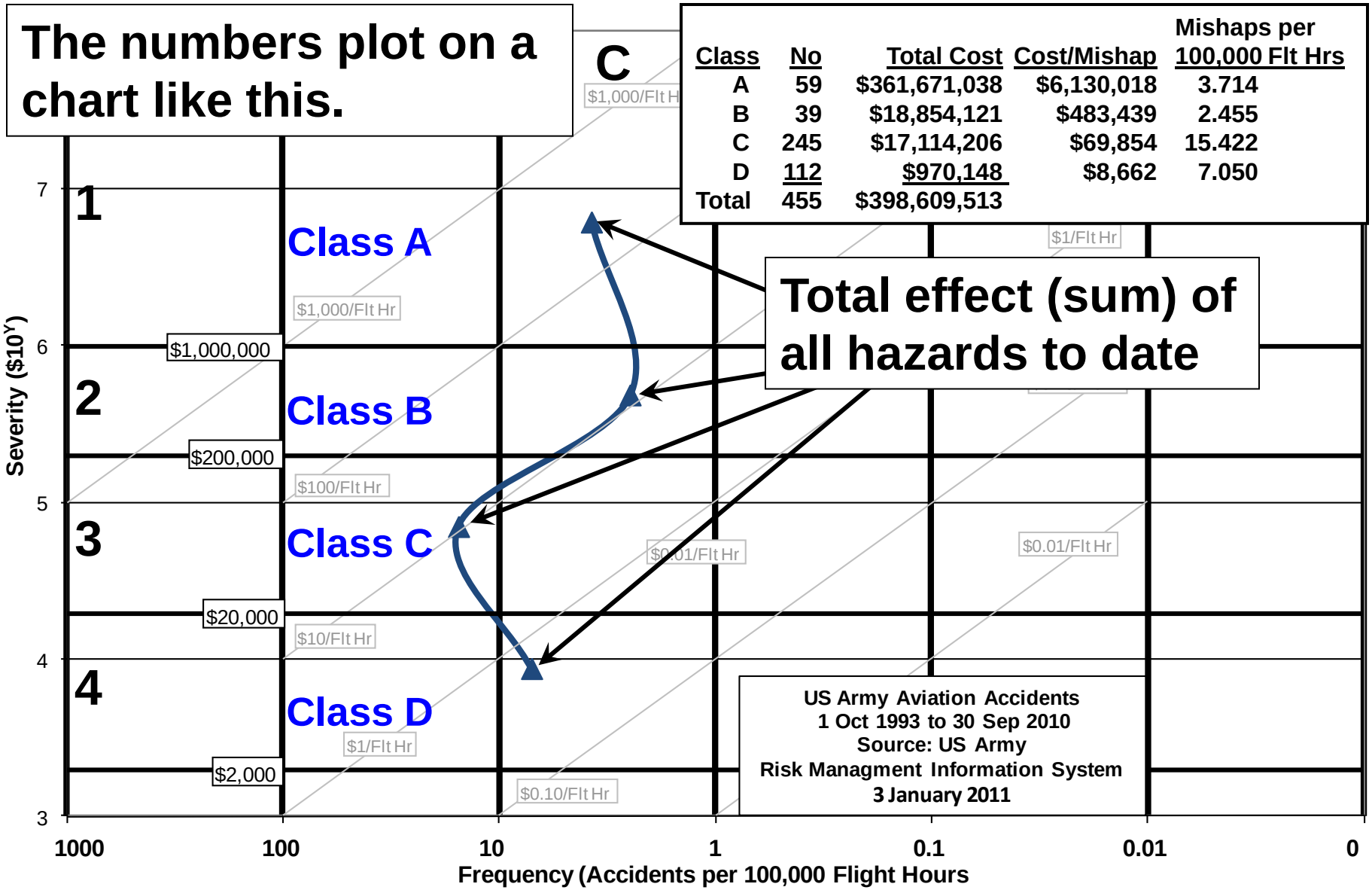
Mishap History

Class	No	Total Cost	Cost/Mishap	Mishaps per 100,000 Flt Hrs
A	59	$361,671,038	$6,130,018	3.714
B	39	$18,854,121	$483,439	2.455
C	245	$17,114,206	$69,854	15.422
D	112	$970,148	$8,662	7.050
Total	455	$398,609,513		

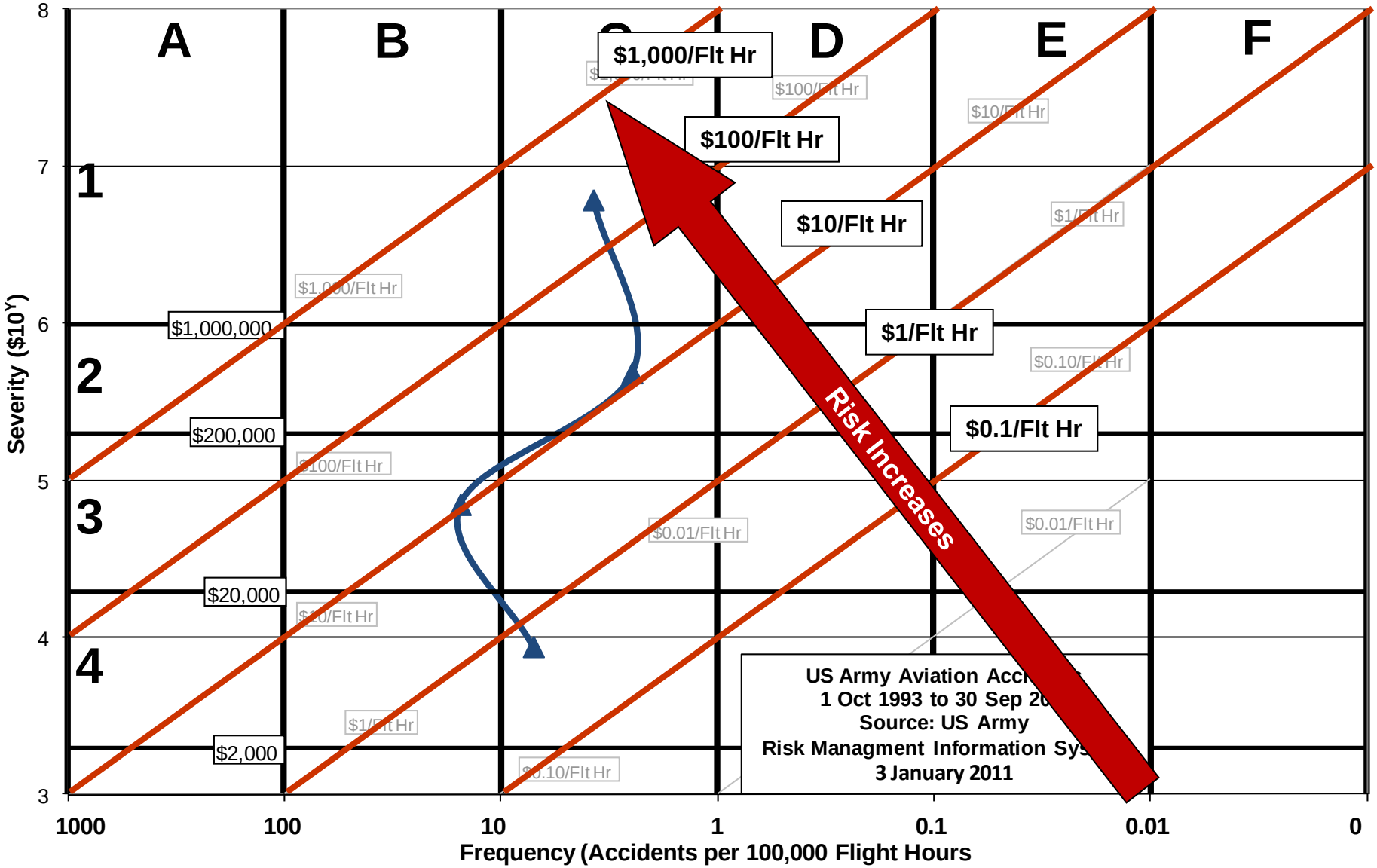
Mishaps

The numbers plot on a chart like this.

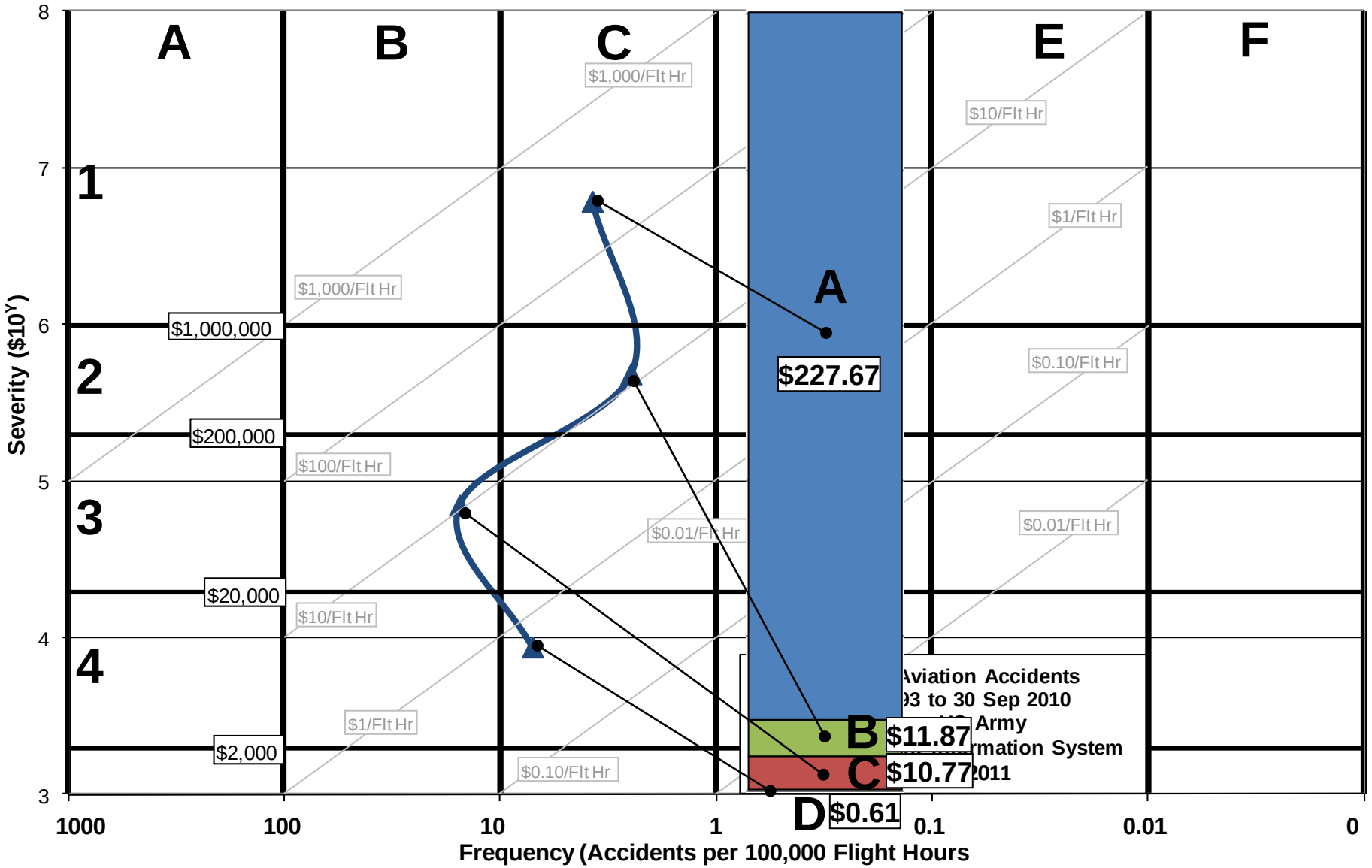
Class	No	Total Cost	Cost/Mishap	Mishaps per 100,000 Flt Hrs
A	59	$361,671,038	$6,130,018	3.714
B	39	$18,854,121	$483,439	2.455
C	245	$17,114,206	$69,854	15.422
D	112	$970,148	$8,662	7.050
Total	455	$398,609,513		



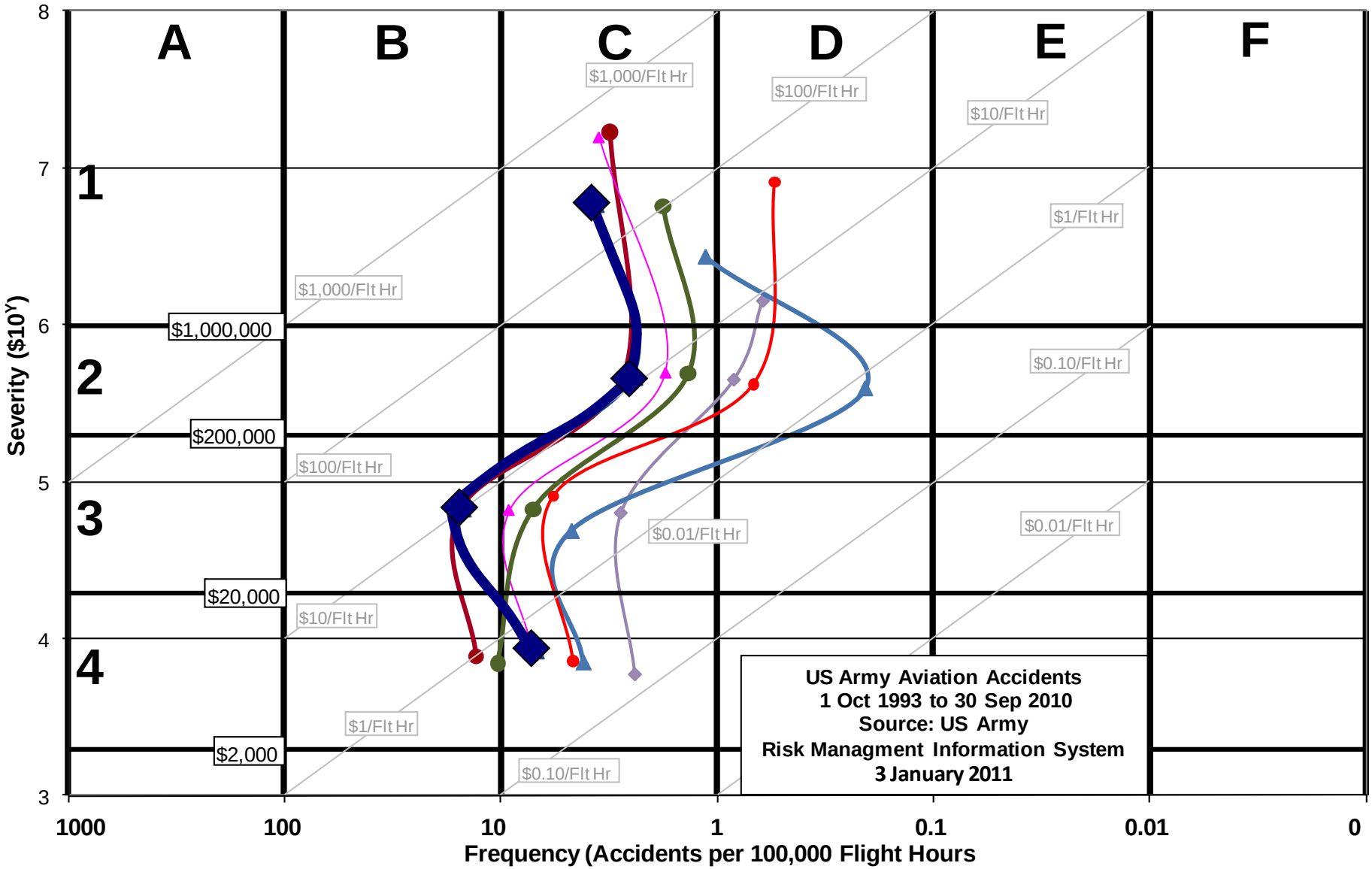
Mishaps


A
B
C
D
E
F
1
2
3
4
$1,000/Flt Hr
$100/Flt Hr
$10/Flt Hr
$1/Flt Hr
$0.1/Flt Hr
Risk Increases
$1,000,000
$200,000
$20,000
$2,000
US Army Aviation Acci
1 Oct 1993 to 30 Sep 20
Source: US Army
Risk Managment Information Sys
3 January 2011
Severity ($10^Y)
8
7
6
5
4
3
1000
100
10
1
0.1
0.01
0
Frequency (Accidents per 100,000 Flight Hours

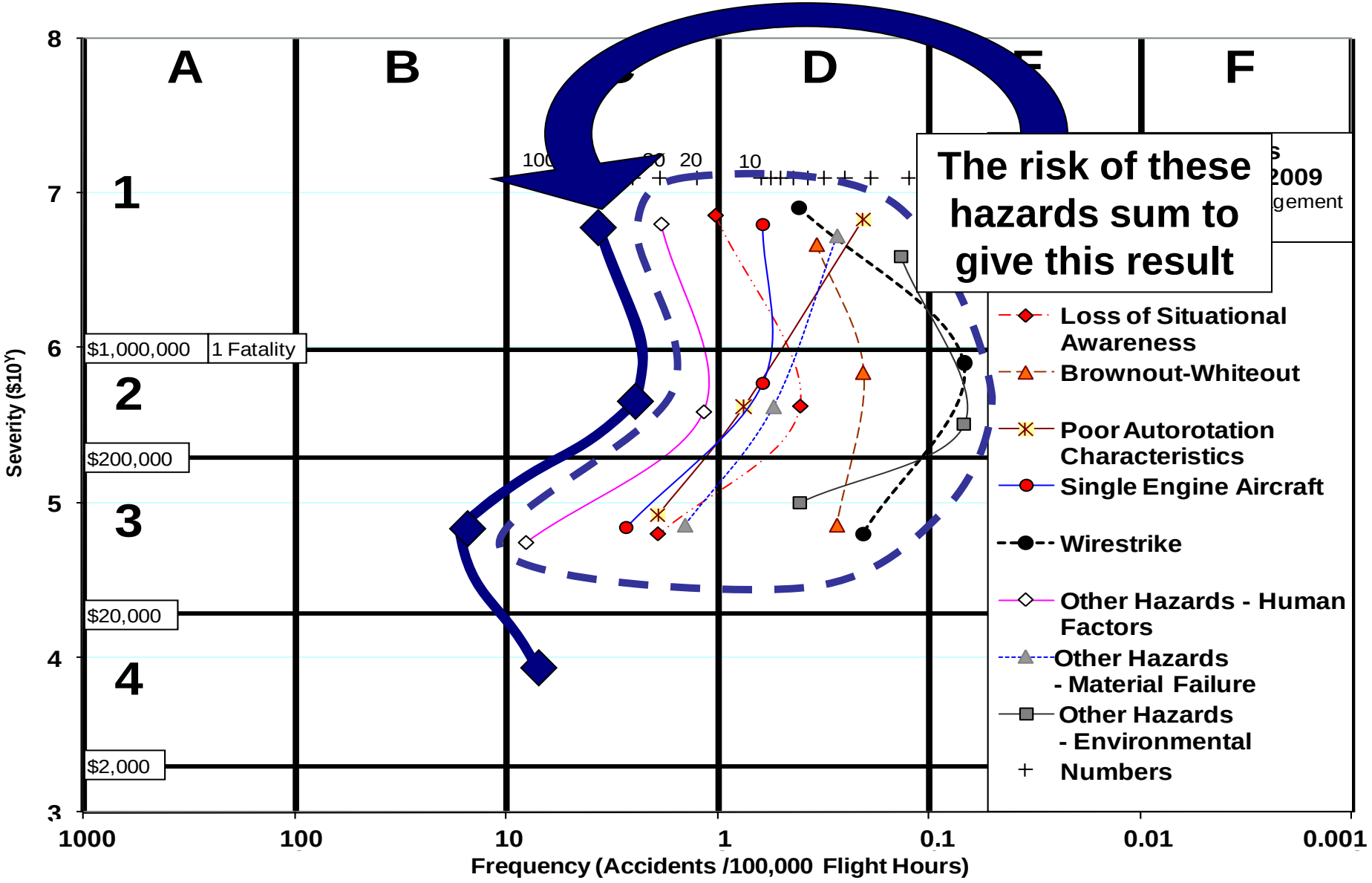
Mishaps


A
B
C
E
F
1
2
3
4
$1,000/Flt Hr
$10/Flt Hr
$1/Flt Hr
$0.10/Flt Hr
$0.01/Flt Hr
$100/Flt Hr
$1,000,000
$200,000
$20,000
$2,000
A
$227.67
B
$11.87
C
$10.77
D
$0.61
Severity ($10^Y)
8
7
6
5
4
3
1000
100
10
1
0.1
0.01
0
Frequency (Accidents per 100,000 Flight Hours
Aviation Accidents
93 to 30 Sep 2010
Army
rmation System
2011

US Army Aviation Mishaps



US Army Aviation Mishaps



Matrix Relative Risk Values (Clemens)

	A	B	C	D	E	F
1	100,000,000	10,000,000	1,000,000	100,000	10,000	1,000
2	10,000,000	1,000,000	100,000	10,000	1,000	100
3	1,000,000	100,000	10,000	1,000	100	10
4	100,000	10,000	1,000	100	10	1

Helo A Hazard Distribution

	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

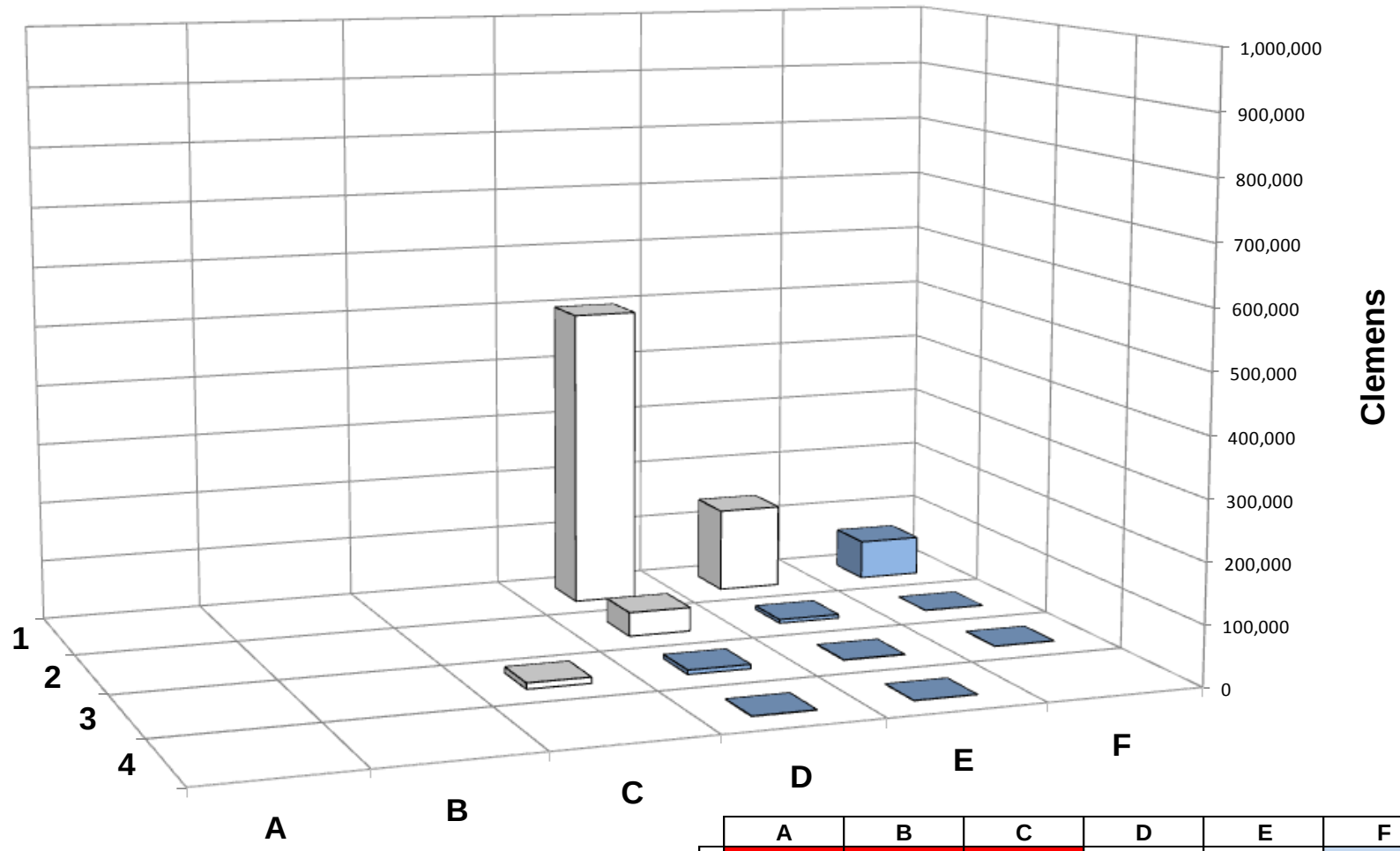
Helo A Matrix
Relative Values (Clemens)

	A	B	C	D	E	F
1				5 x 100,000 = 500,000	14 x 10,000 = 140,000	65 x 1,000 = 65,000
2				4 x 10,000 = 40,000	6 x 1,000 = 6,000	2 x 100 = 200
3			1 x 10,000 = 10,000	7 x 1,000 = 7,000	5 x 100 = 500	4 x 10 = 40
4				2 x 100 = 200	1 x 10 = 10	

Helo A Matrix
Relative Values (Clemens)

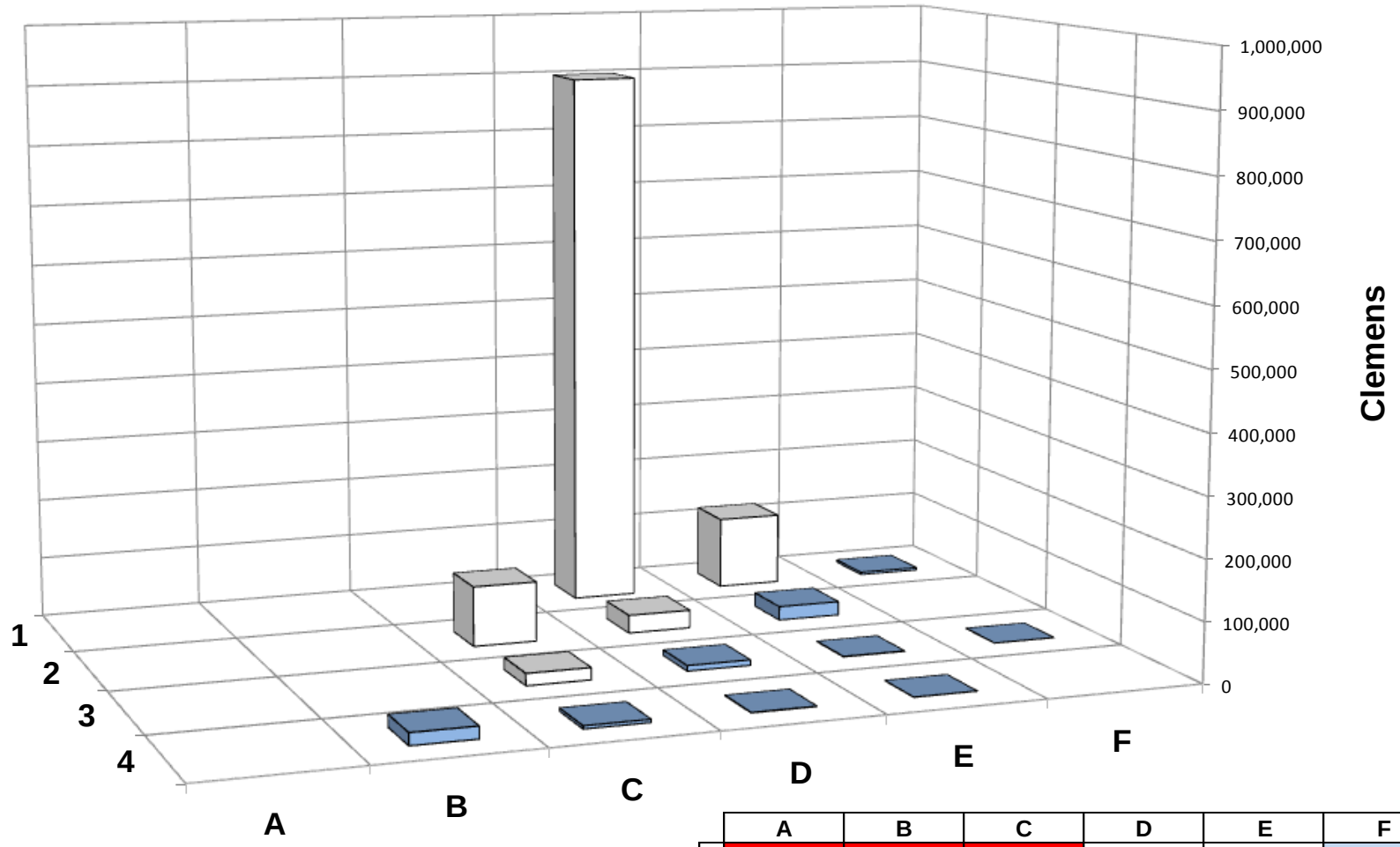
	A	B	C	D	E	F
1				500,000	140,000	65,000
2				40,000	6,000	200
3			10,000	7,000	500	40
4				200	10	

Helicopter A



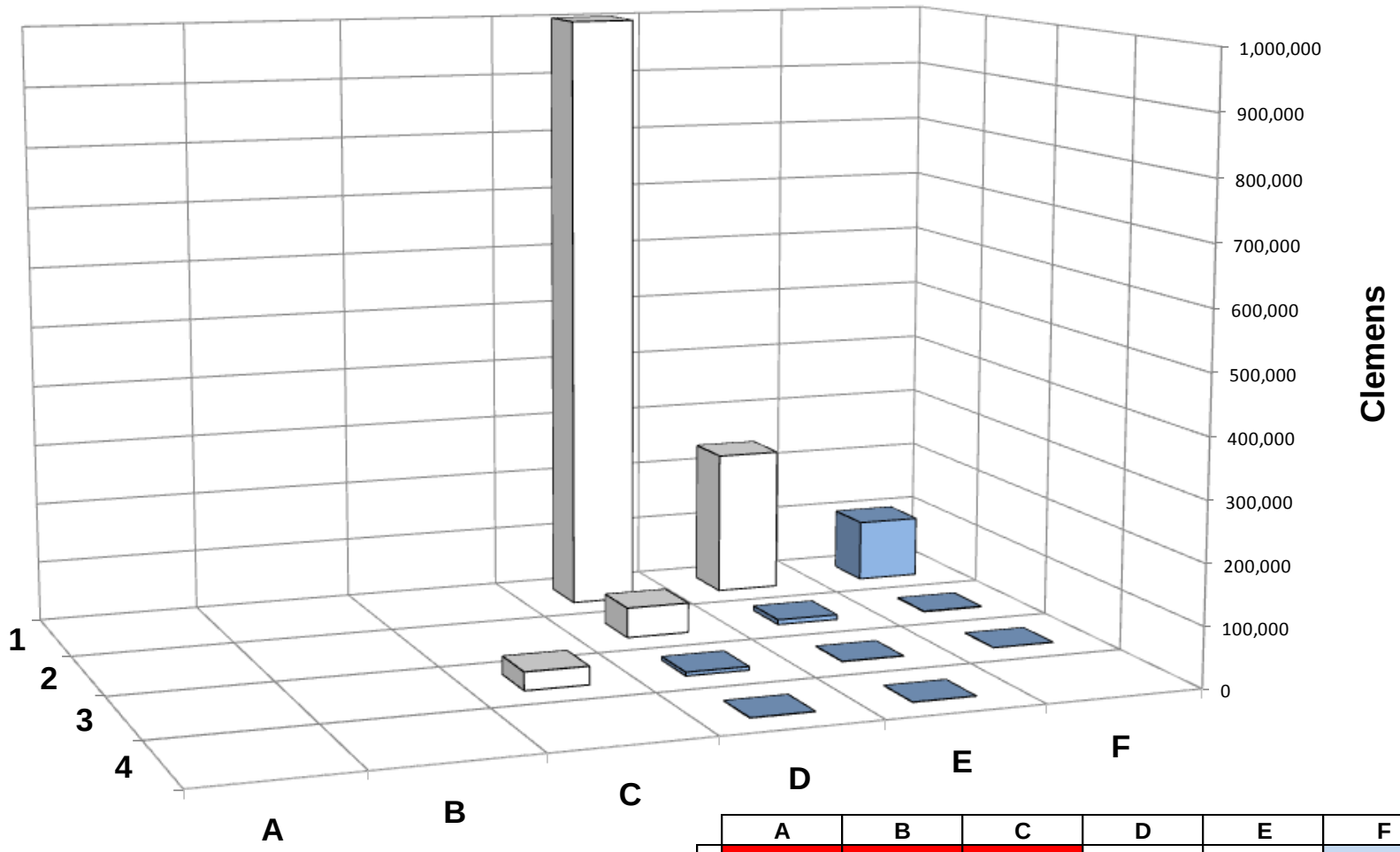
	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Helicopter B



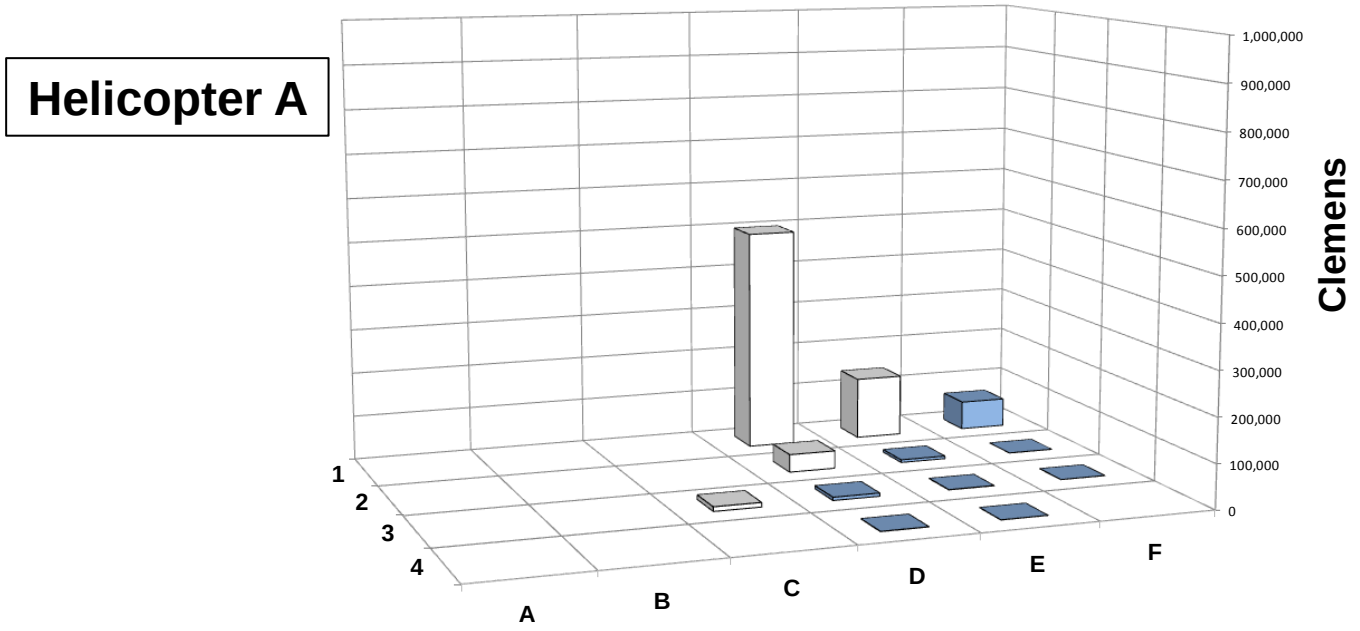
	A	B	C	D	E	F
1				9	12	4
2			1	3	23	
3			2	9	6	1
4		2	5	5	2	

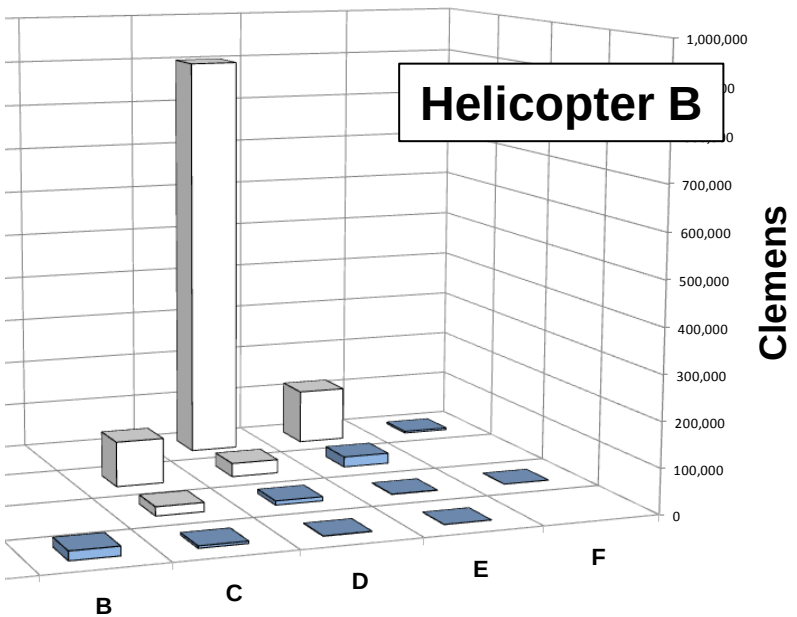
Helicopter C

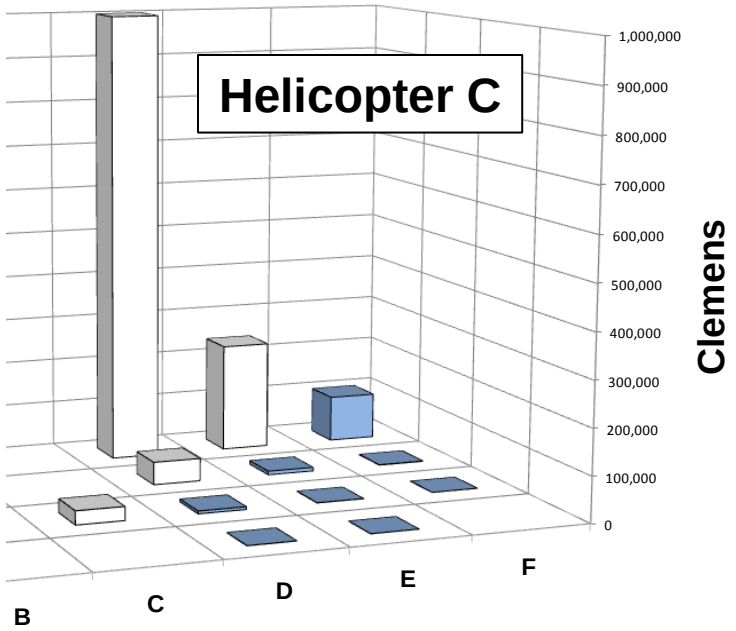


	A	B	C	D	E	F
1				10	24	102
2				5	8	8
3			3	6	3	2
4				1	1	

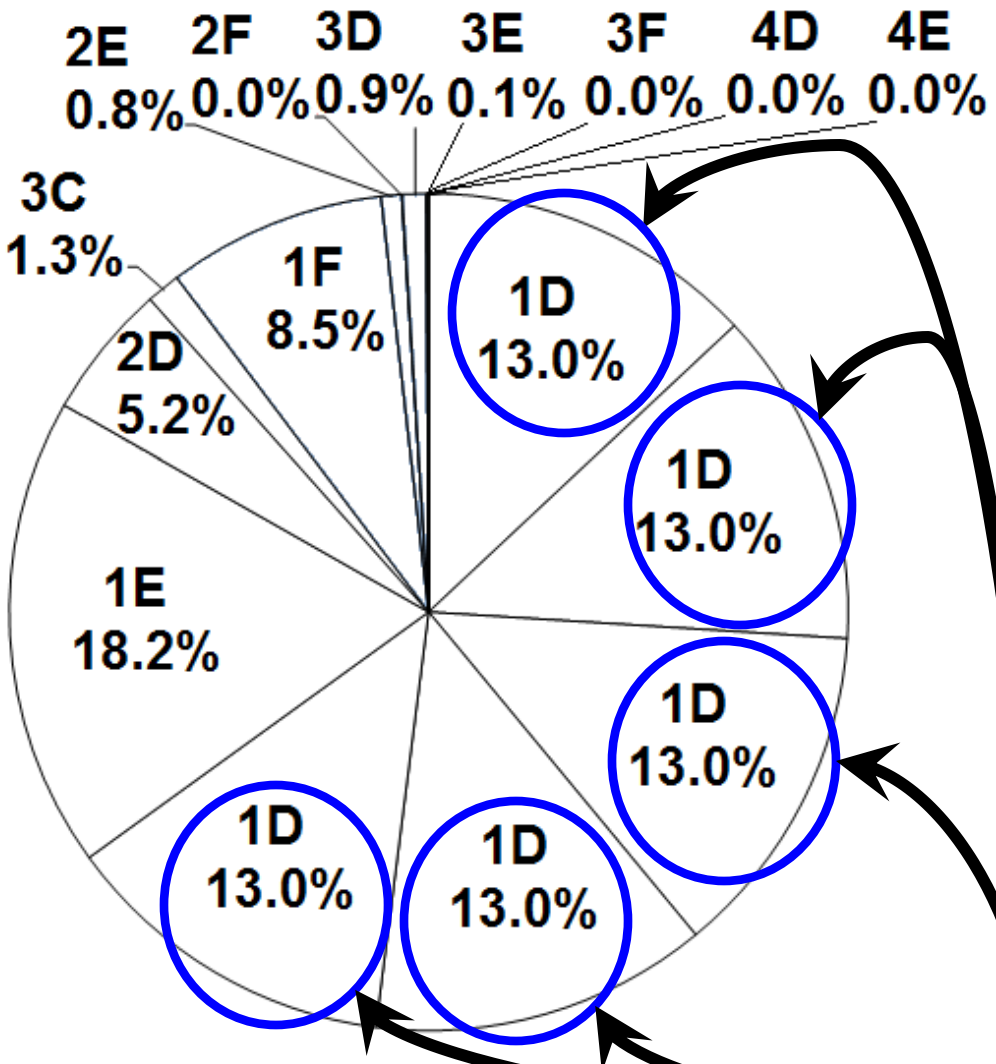
Side by Side Relative Risk by RAC





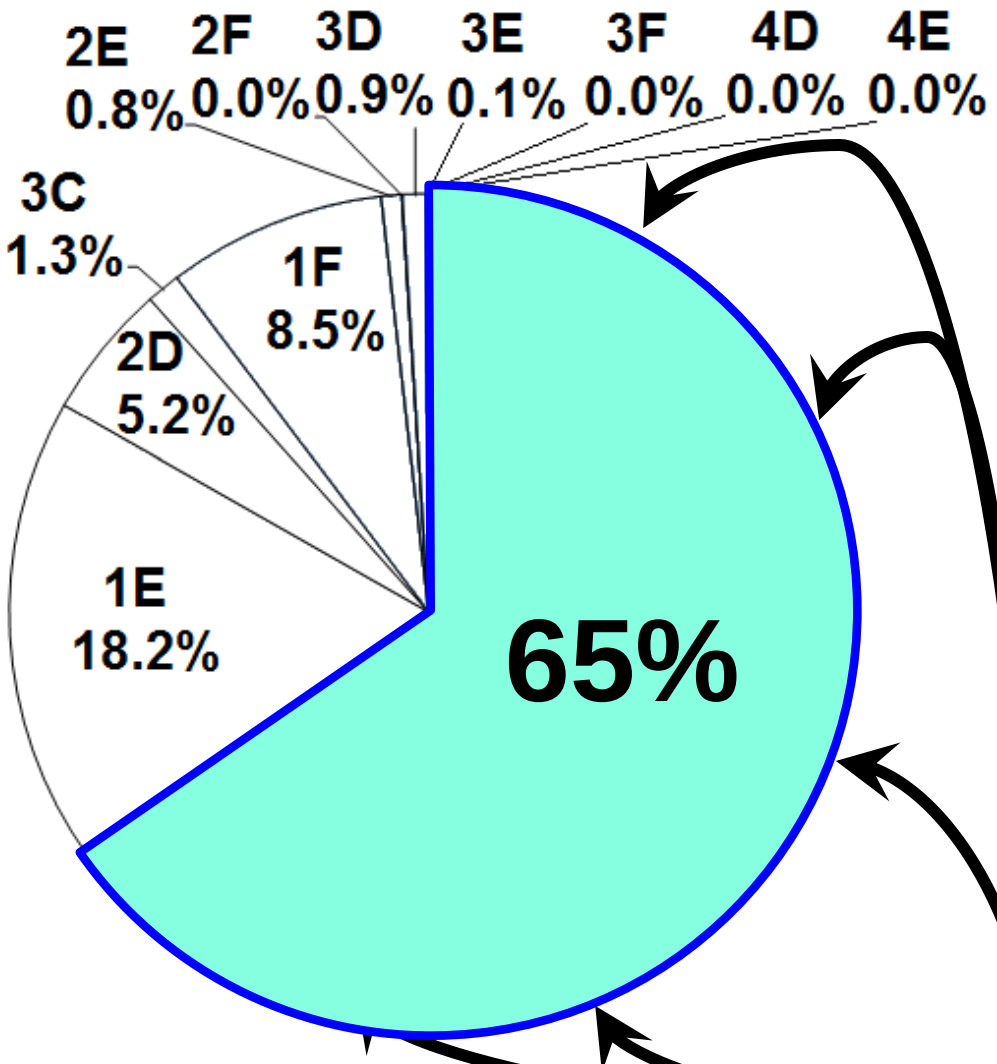


Risk Pie Chart by RAC



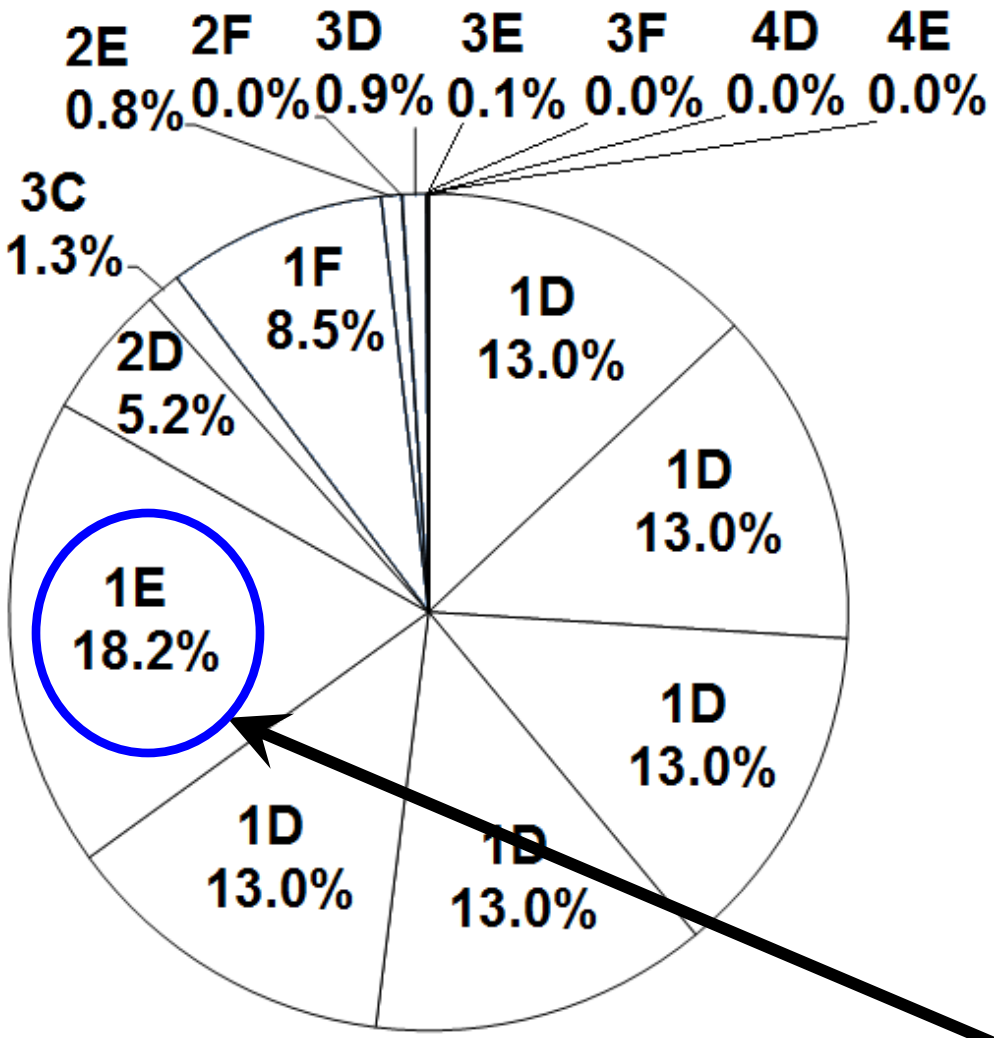
	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Risk Pie Chart by RAC



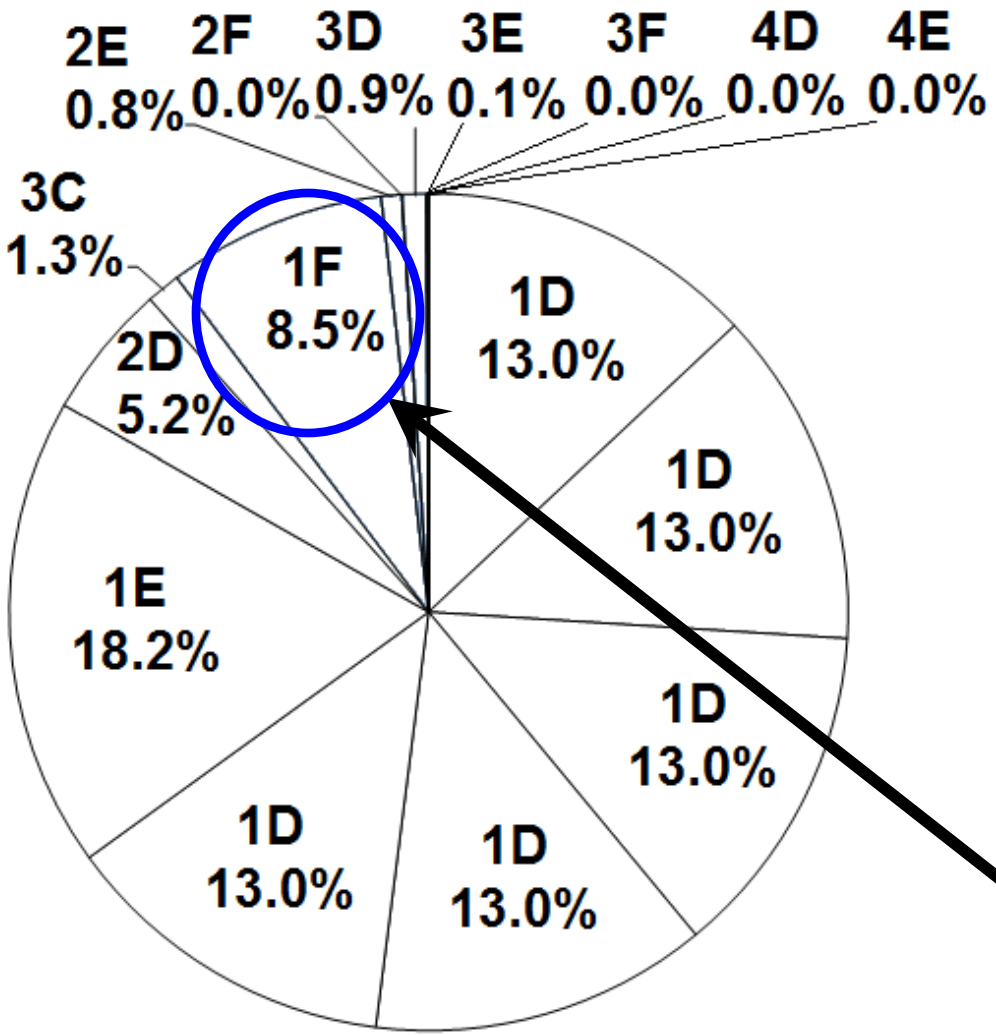
	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Risk Pie Chart by RAC



	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

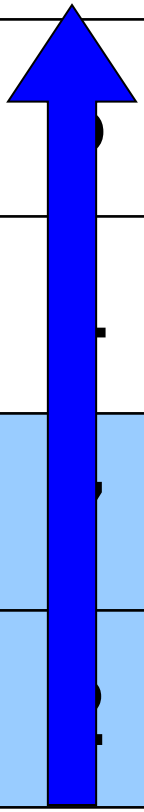
Risk Pie Chart by RAC



	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Hazard Risk Profile

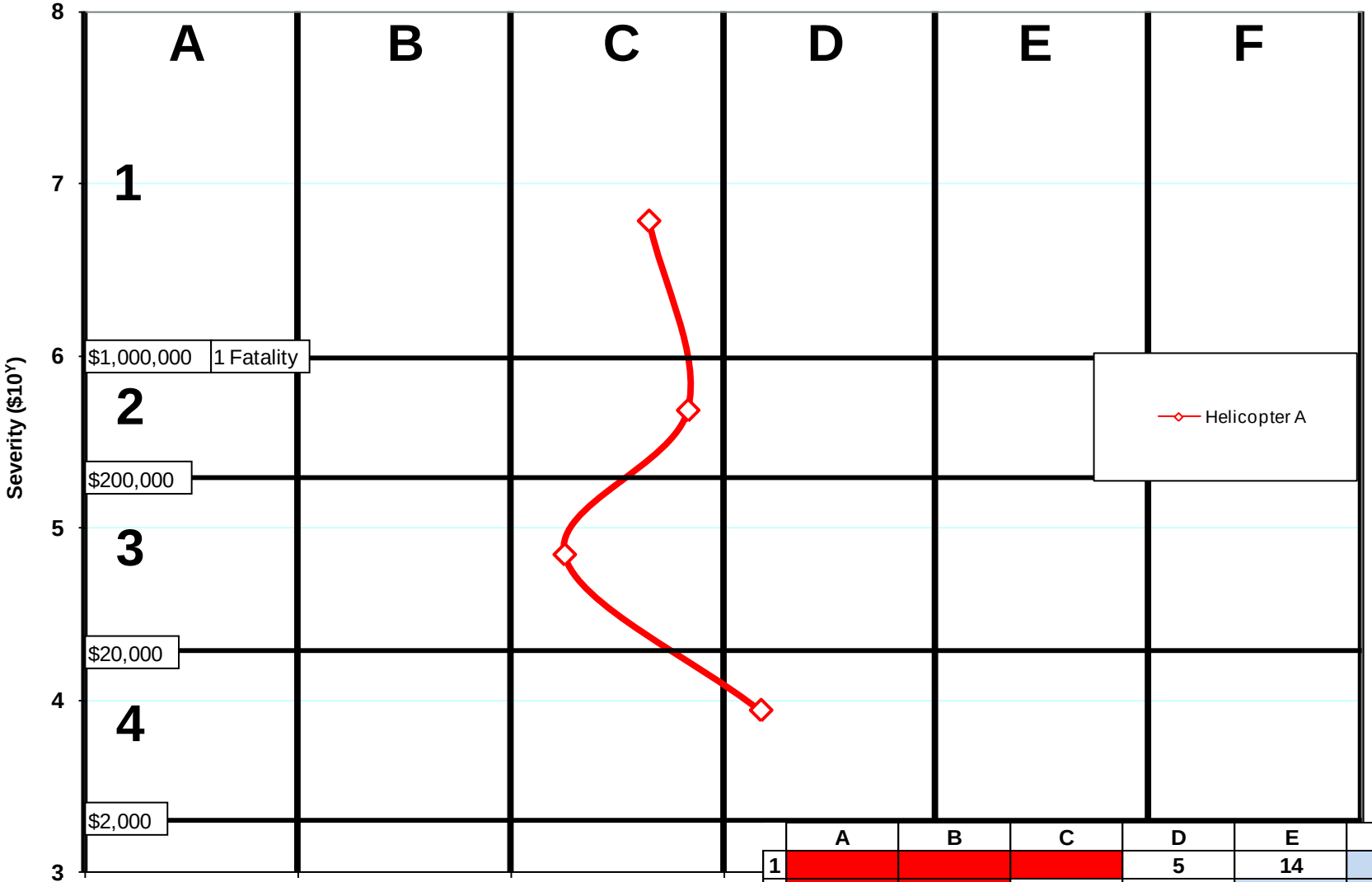
		3.16E-04	3.16E-05	3.16E-06	3.16E-07	3.16E-08
	A	B	C	D	E	F
1					14	65
2					6	2
3			1		5	4
4					1	



Hazard Risk Profile

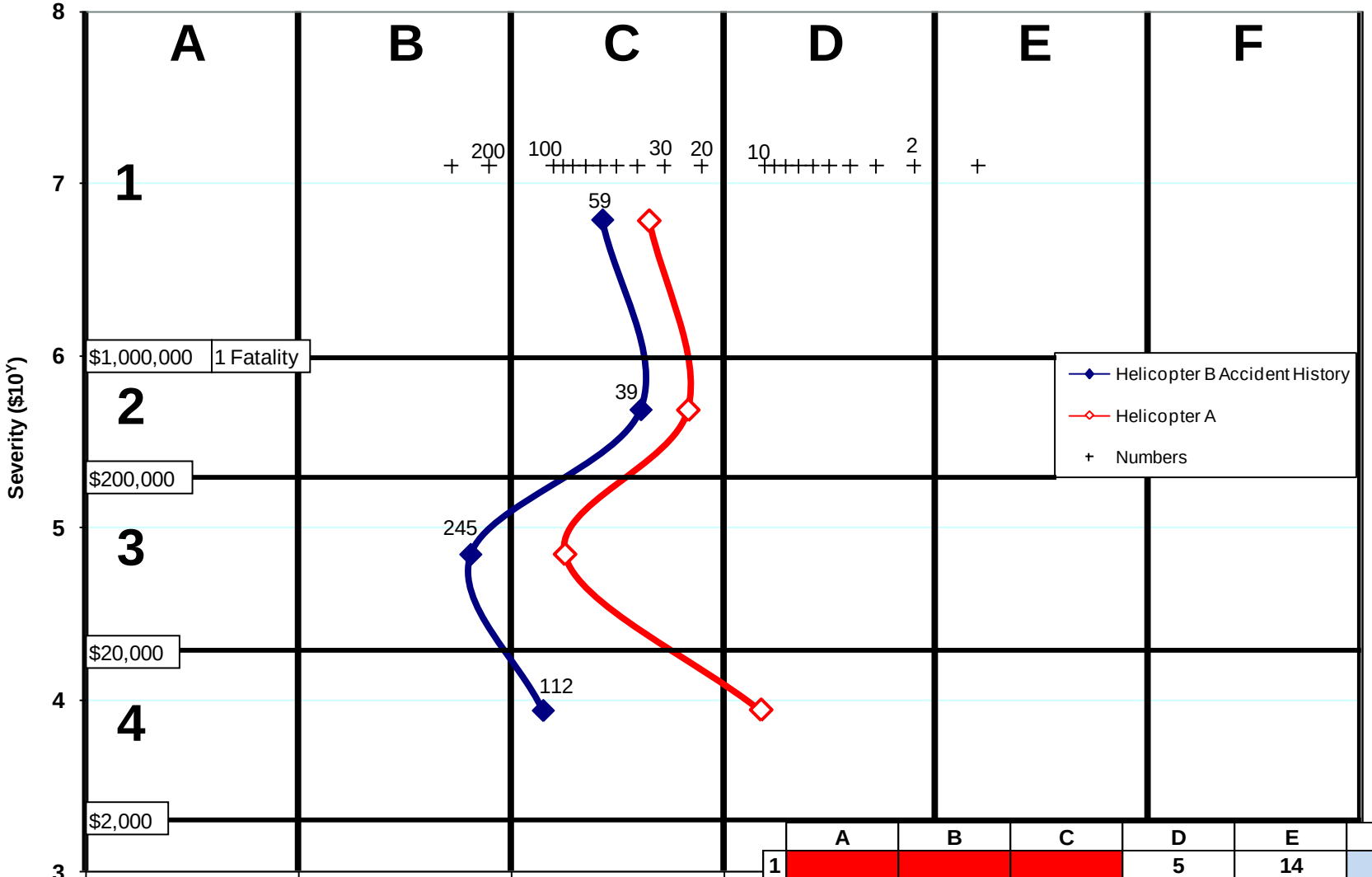
			3.16E-04	3.16E-05	3.16E-06	3.16E-07	3.16E-08
	A	B	C	D	E	F
1	2.23E-05	Sum		5 x 3.16E-06 = 1.58E-05	14 x 3.16E-07 = 4.43E-06	65 x 3.16E-08 = 2.06E-06
2	1.46E-05	Sum		4 x 3.16E-06 = 1.26E-05	6 x 3.16E-07 = 1.90E-06	2 x 3.16E-08 = 6.32E-08
3	5.55E-05	Sum	1 x 3.16E-05 = 3.16E-05	7 x 3.16E-06 = 2.21E-05	5 x 3.16E-07 = 1.58E-06	4 x 3.16E-08 = 1.26E-07
4	6.64E-06	Sum		2 x 3.16E-06 = 6.32E-06	1 x 3.16E-07 = 3.16E-07	

Hazard Risk Profile



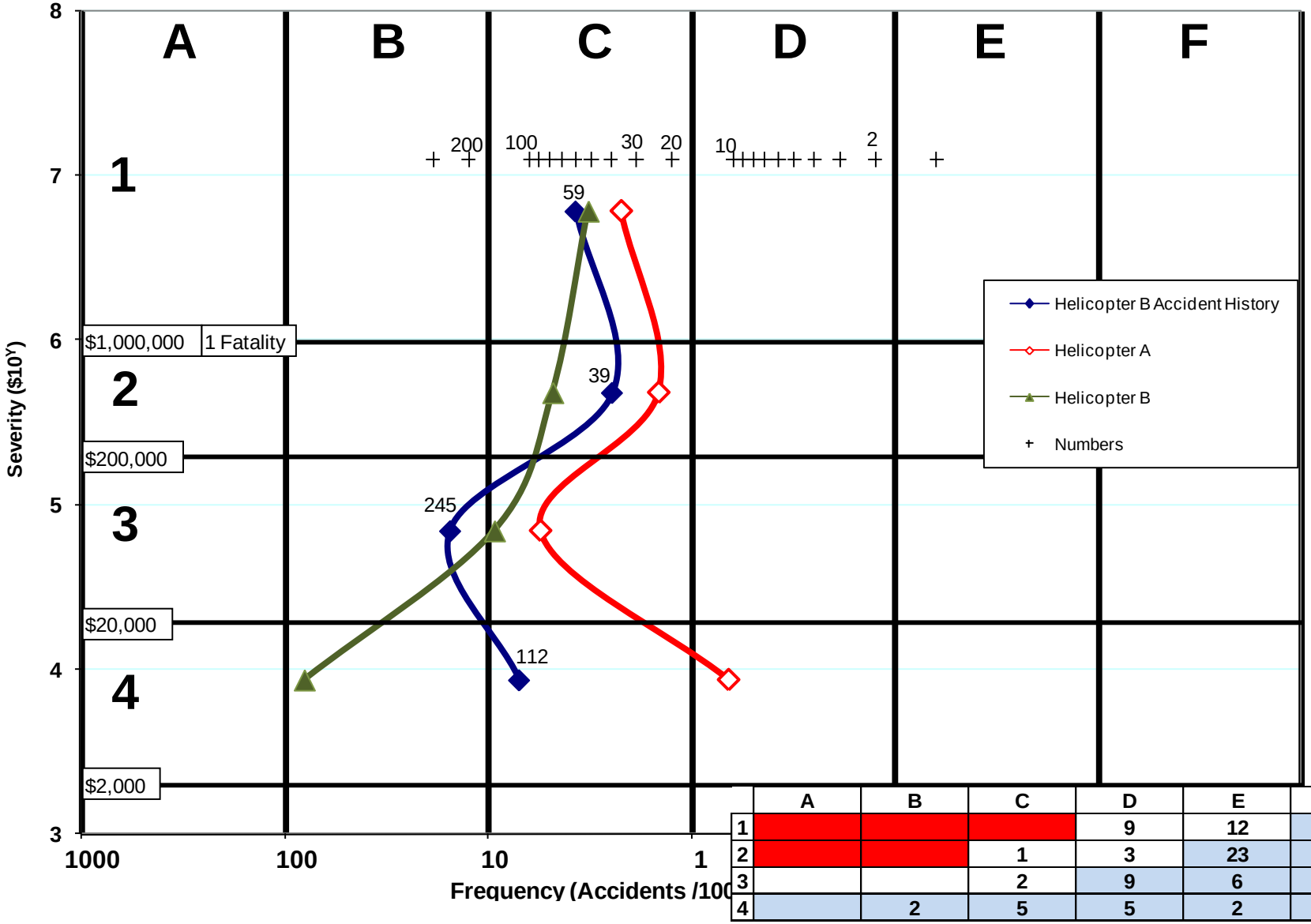
	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Comparing Hazard Profile to Accident History



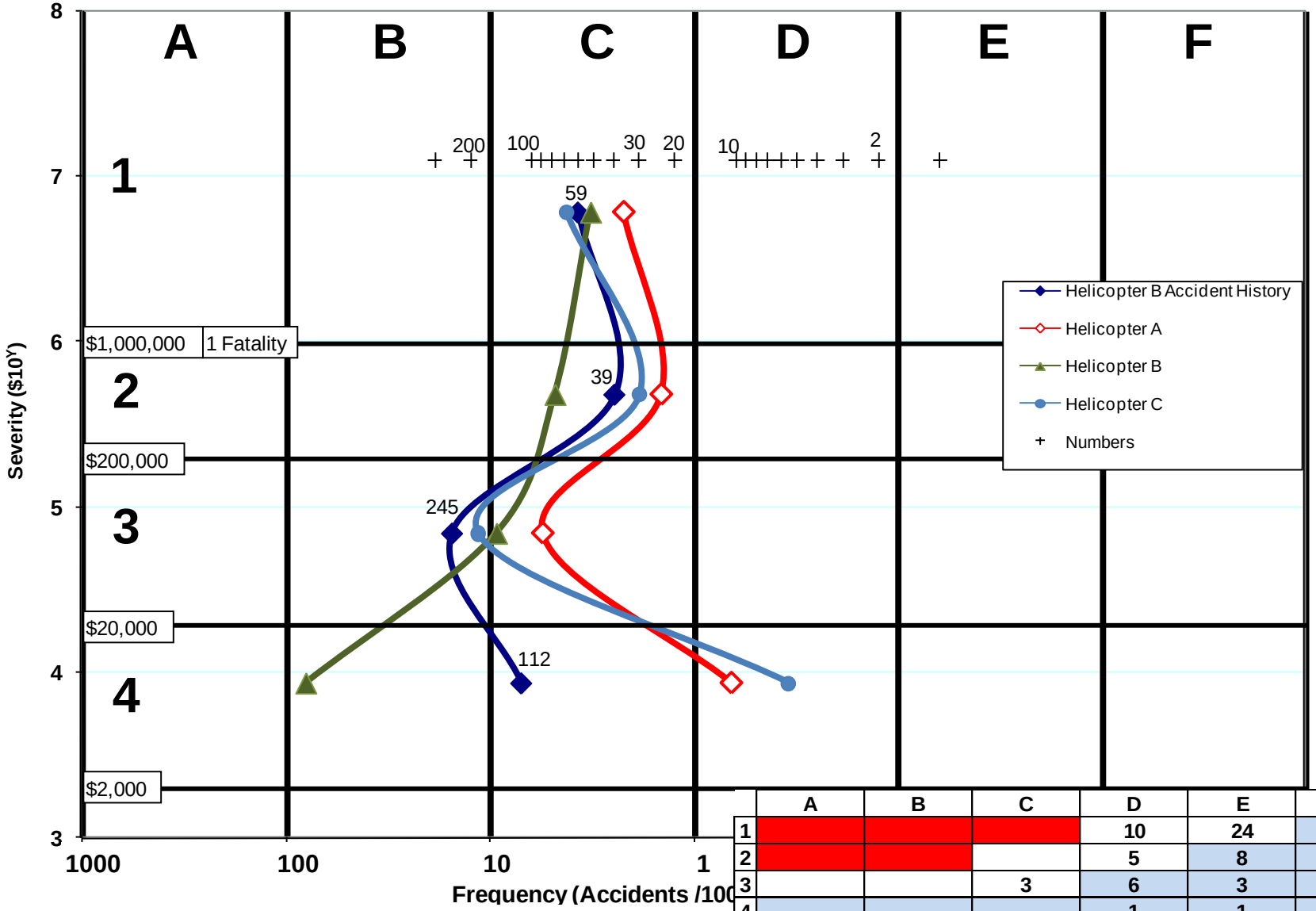
	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Comparing Hazard Profile to Accident History



	A	B	C	D	E	F
1				9	12	4
2			1	3	23	
3			2	9	6	1
4		2	5	5	2	

Comparing Hazard Profile to Accident History



	A	B	C	D	E	F
1				10	24	102
2				5	8	8
3			3	6	3	2
4				1	1	

Summary

Attributes of a well-designed risk assessment matrix

- Severity scale covers full range of possible outcomes
- Probability calibrated with reference to an exposure interval
- Equally proportioned, logarithmic scales (1, 10, 100, 1000…)
- Cartesian Orientation – Increase up and to the right
- Risk levels assigned to cells consistent with contours of equal risk (iso-risk contours)

Severity		1	2	3	4	5	6	7	8
		≥$2k	≥$20k	≥$200k	≥$2M	≥$20M	≥$200M	≥$2B	≥$20B
Frequency		Injury, no lost work day	Lost Work Day	Permanent partial disability	≥1 Fatality	≥10 Fatalities	≥100 Fatalities	≥1,000 Fatalities	≥10,000 Fatalities
A	>100								
B	>10								
C	>1								
D	>0.1								
E	>0.01								
F	>0.001								
G	>0.0001								
H	>0.00001								
I	> 0.000001								
J	≤ 0.000001								

Prohibitive SECDEF

High - CAE

Serious - PEO

Medium - PM

Low – SSWG/Principal for Safety

Summary

Attributes of a well-designed risk assessment matrix

- Sufficient probability or frequency categories so highest severity level can be assessed at the PM level of risk if the probability or frequency of occurrence is low enough
- A risk assessment code for hazards whose risk has been eliminated
- Easily tailored with reporting of risk consistent with other systems within the family of systems.

Severity		1	2	3	4	5	6	7	8
		≥$2k	≥$20k	≥$200k	≥$2M	≥$20M	≥$200M	≥$2B	≥$20B
Frequency		Injury, no lost work day	Lost Work Day	Permanent partial disability	≥1 Fatality	≥10 Fatalities	≥100 Fatalities	≥1,000 Fatalities	≥10,000 Fatalities
A	>100								
B	>10								
C	>1						Prohibitive SECDEF		
D	>0.1					High - CAE			
E	>0.01				Serious - PEO				
F	>0.001			Medium - PM					
G	>0.0001	Low – SSWG/Principal for Safety							
H	>0.00001								
I	> 0.000001								
J	≤ 0.000001								

Summary Understanding Probability

Math Definition:

- **Repeat a random experiment "n" number of times.**
- **If a specific outcome has occurred "f" times in these n trials, the number "f" is the frequency of the outcome.**
- **The ratio f/n is the relative frequency of the outcome.**
- **A relative frequency is usually very unstable for small values of "n," but it tends to stabilize about some number "p" as "n" increases.**
- **The number "p" is the probability of the outcome.**

$$p = f / n$$

for very large values of n

Simple example:

Probability of rolling a "3" with one die.

Roll #1 - "5", f/n = 0

Roll #2 - "2", f/n = 0

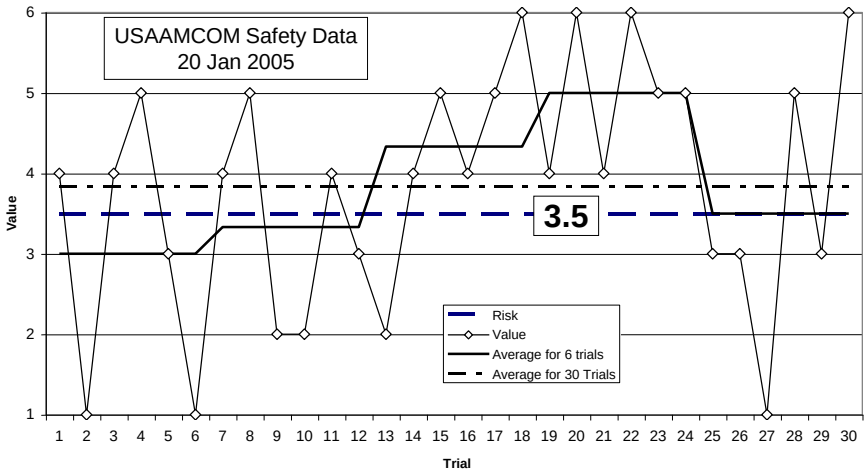
Roll #3 - "3", f/n = 1/3 = .333…

Roll #4 - "4", f/n = 1/4 = .25

Roll #1,000: 163 "3"s, f/n = 163/1000 = .163

Rolls approach infinity f/n = .166666….

Roll a single die 30 times. The expected value of each roll is 3.5. What you actually get is somewhat different.



Hazard: AH-64 strikes wire results in Class A mishap

Probability: 4.406E-06 occurrences per flight hour

1 Flight Hr, no mishap, rate = 0

1,000 Flight Hrs, no mishap, rate = 0

176,182 Flight Hrs, 1 mishap, rate = 5.676E-06 /flt hr

274,539 Flight Hrs, 2 mishaps, rate = 7.285E-06 /flt hr

700,462 Flt Hrs, 3 mishaps, rate = 4.283E-06 /flt hr

10,000,000 Flt Hrs, 46 mishaps, rate = 4.600E-06 /flt hr

1,000,000,000 Hrs, 4407 mishaps, rate = 4.407E-06 /flt hr

Flight hours approach infinity, rate = 4.406E-06 /flt hr

Summary Expanded Matrix

Applying Probability Classifications to a military helicopter

Fleet Size = 368 aircraft

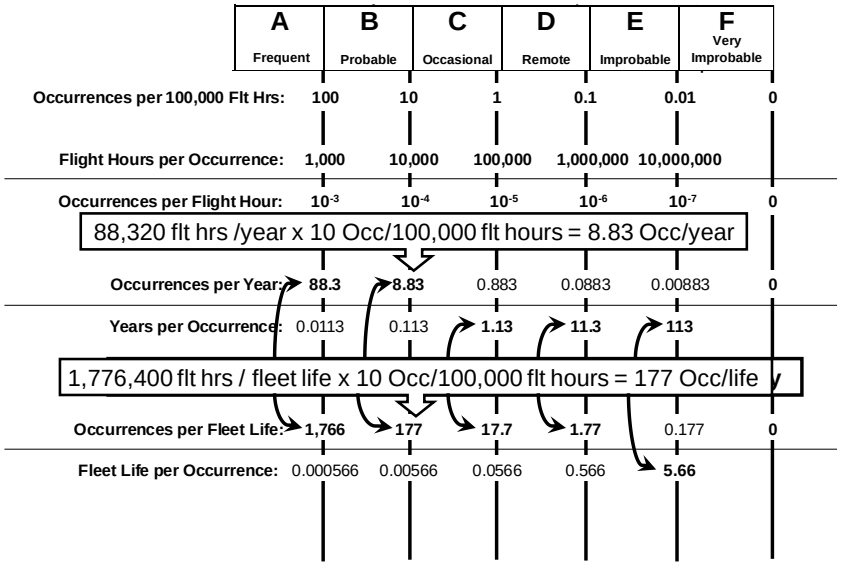
Utilization = 240 hours/year

Life = 20 years/aircraft

Aircraft Life = 240 x 20
= 4,800 hours

Fleet Exposure Hours = 368 x 240 x 20
= 1,776,400 hours

Fleet Hours per Year = 368 x 240
= 88,320 hours



Assumptions

Fleet Size: 368 aircraft
Utilization: 240 hours/yr
Aircraft Life: 20 years
Aircraft Exposure Hours: 4,800 hours
Fleet Exposure Hours: 1,766,400 hours
Fleet Hours per Year: 88,320 hours

	A Frequent	B Probable	C Occasional	D Remote	E Improbable	F Very Improbable	0R
Occurrences per 100,000 Flt Hrs	100	10	1	0.1	0.01	0	
Flight Hours per Occurrence	1,000	10,000	100,000	1,000,000	10,000,000		
Occurrences per Flight Hour	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	0	
Fleet Life							
Occurrences per Year	88.3	8.83	0.883	0.0883	0.00883	0	
Years per Occurrence	0.0113	0.113	1.13	11.3	113		
Occurrence per Fleet Life	1,766	177	17.7	1.77	0.177	0	
Fleet Life per Occurrence	0.000566	0.00566	0.0566	0.566	5.66		

Catastrophic 1 ($1M)	1A (High CSA/AAE)	1B	1C	1D	1E	1F	
Critical 2 ($200K)	2A	2B	2C (Medium PEO)	2D	2E (Low PM)	2F	
Marginal 3 ($20K)	3A	3B	3C	3D	3E	3F	
Negligible 4	4A	4B	4C	4D	4E	4F	

Summary

Accidents on a Matrix

Based on this relationship between mishap risk and mishap loss, we can plot mishap histories on a risk matrix as follows:

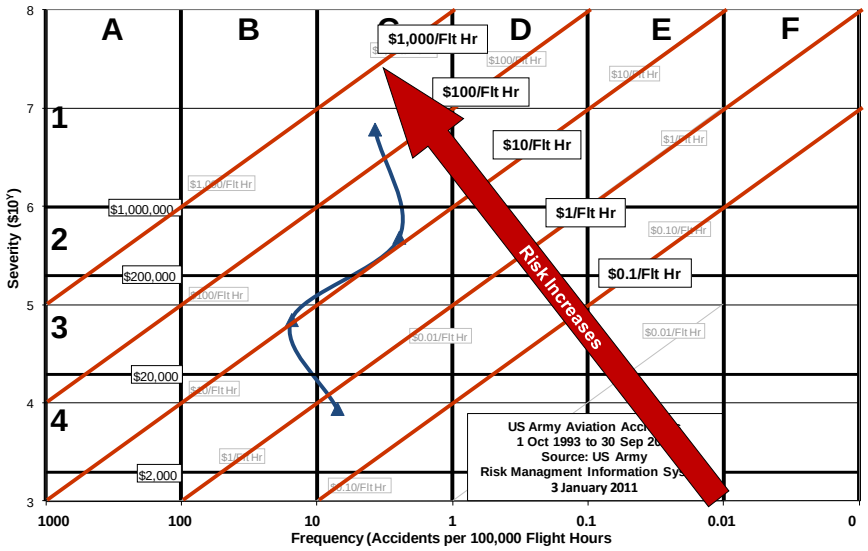
$$\text{Severity} = \frac{\text{Total Cost from Class A mishaps}}{\text{Total Number of Class A mishaps}}$$

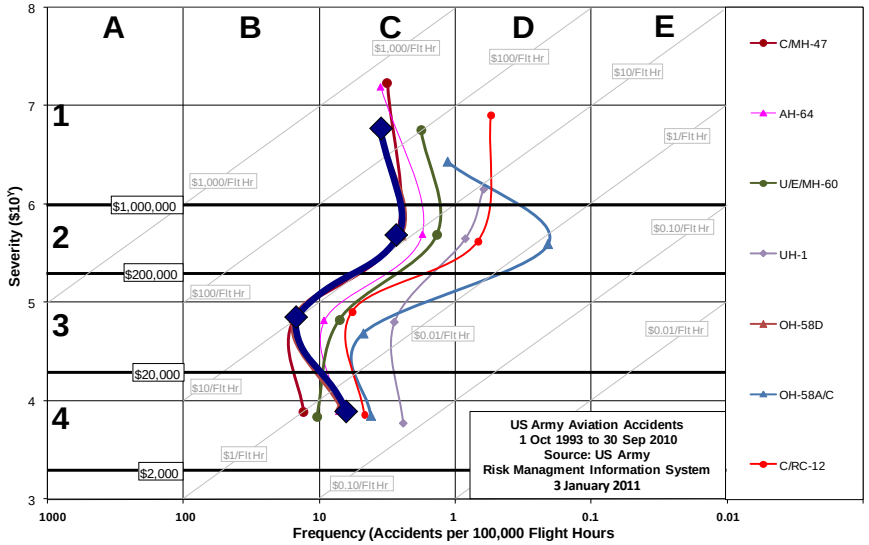
$$= \frac{\$1{,}305{,}079{,}886}{83} = \$\ 15{,}723{,}854$$

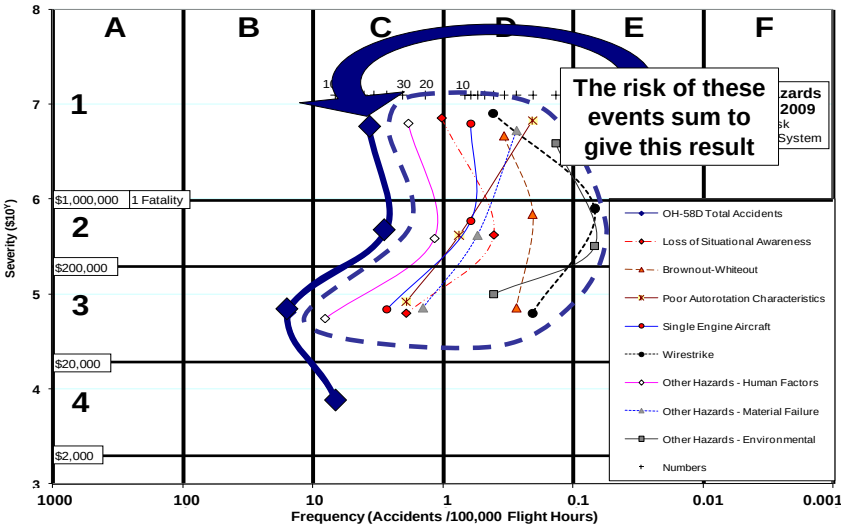
$$\text{Probability} = \frac{\text{Total Number of Class A mishaps}}{\text{Total Hours Flown}}$$

$$= \frac{83}{2{,}351{,}860} = 3.529 \text{ mishaps / 100,000 Flt Hrs}$$

21







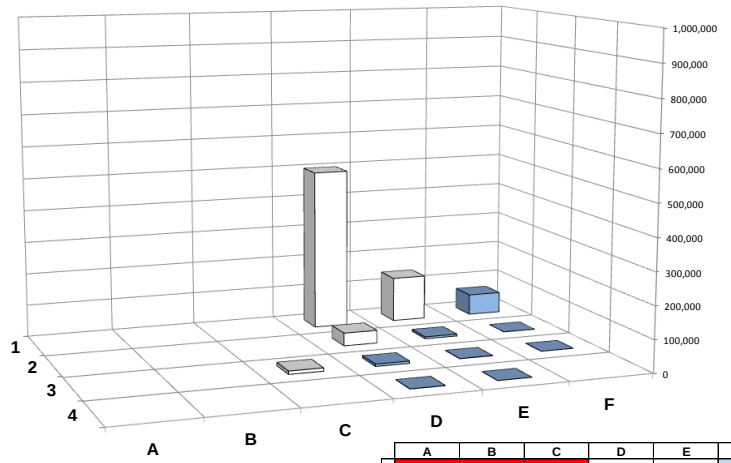
Summary

	A	B	C	D	E	F
1	100,000,000	10,000,000	1,000,000	100,000	10,000	1,000
2	10,000,000	1,000,000	100,000	10,000	1,000	100
3	1,000,000	100,000	10,000	1,000	100	10
4	100,000	10,000	1,000	100	10	1

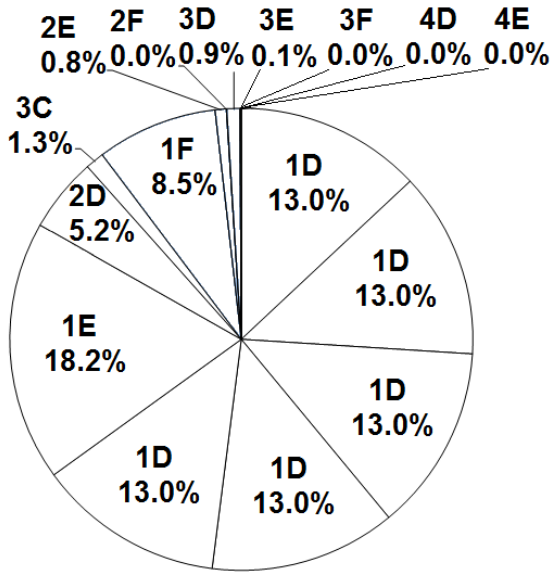
Relative Risk Values (Clemens)

	A	B	C	D	E	F
1				500,000	140,000	65,000
2				40,000	6,000	200
3			10,000	7,000	500	40
4				200	10	

Helicopter A



	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	



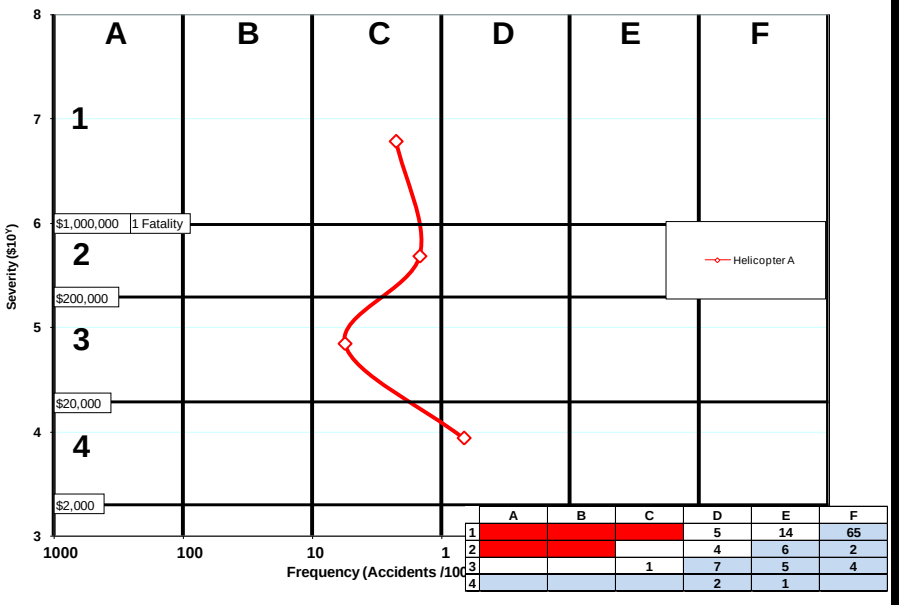
	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	

Summary

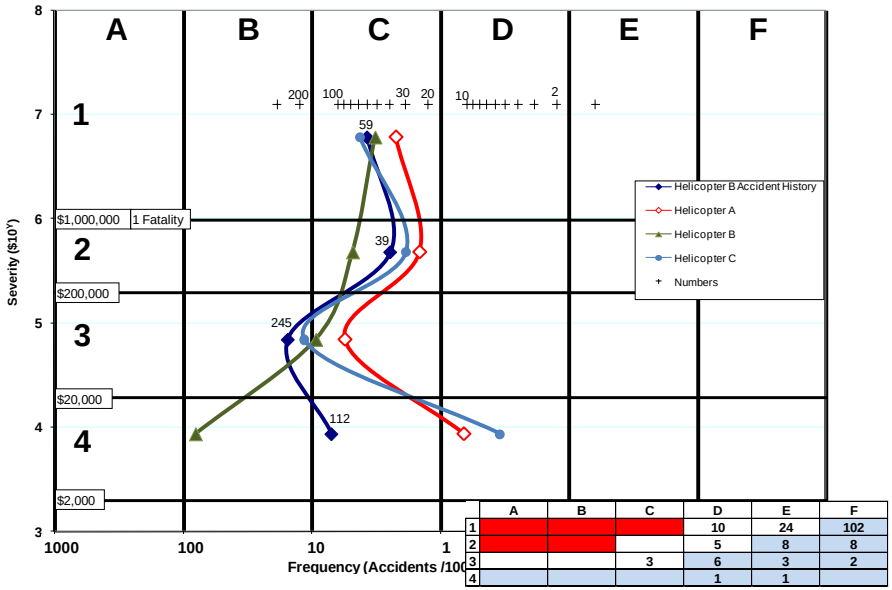
Hazard Risk Profile

	3.16E-04 A	3.16E-05 B	3.16E-06 C	3.16E-07 D	3.16E-08 E	F
1					14	65
2					6	2
3			1		5	4
4					1	

	A	3.16E-04 B	3.16E-05 C	3.16E-06 D	3.16E-07 E	3.16E-08 F
1	2.23E-05	Sum		5 x 3.16E-06 = 1.58E-05	14 x 3.16E-07 = 4.43E-06	65 x 3.16E-08 = 2.06E-06
2	1.46E-05	Sum		4 x 3.16E-06 = 1.26E-05	6 x 3.16E-07 = 1.90E-06	2 x 3.16E-08 = 6.32E-08
3	5.55E-05	Sum	1 x 3.16E-05 = 3.16E-05	7 x 3.16E-06 = 2.21E-05	5 x 3.16E-07 = 1.58E-06	4 x 3.16E-08 = 1.26E-07
4	6.64E-06	Sum		2 x 3.16E-06 = 6.32E-06	1 x 3.16E-07 = 3.16E-07	



	A	B	C	D	E	F
1				5	14	65
2				4	6	2
3			1	7	5	4
4				2	1	



	A	B	C	D	E	F
1				10	24	102
2				5	8	8
3			3	6	3	2
4				1	1	

Conclusions

- High degree of precision? – No
- Gets hazards to appropriate cell of the matrix
- Confidence that overall assessment ≈ reality
- Very useful for programs with:
  - Reasonably good accident data for analysis
  - A well-designed matrix
- Just one of many tools for managing system safety risk

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Questions?